

## **An Analysis of Amarillo's Rainfall Totals**

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Amarillo's local weather station has been measuring and reporting rainfall totals since 1892. In this report we will look at cumulative distributions for both annual and monthly rainfall totals and develop mathematical models for those distributions.

### **Part 1. Annual rainfall totals**

Annual rainfall totals in Amarillo have ranged from 7.01 inches (2011) to 39.75 inches (1923). The annual rainfall data, consisting of 133 values, can be found at the NOAA Online Weather Data website.

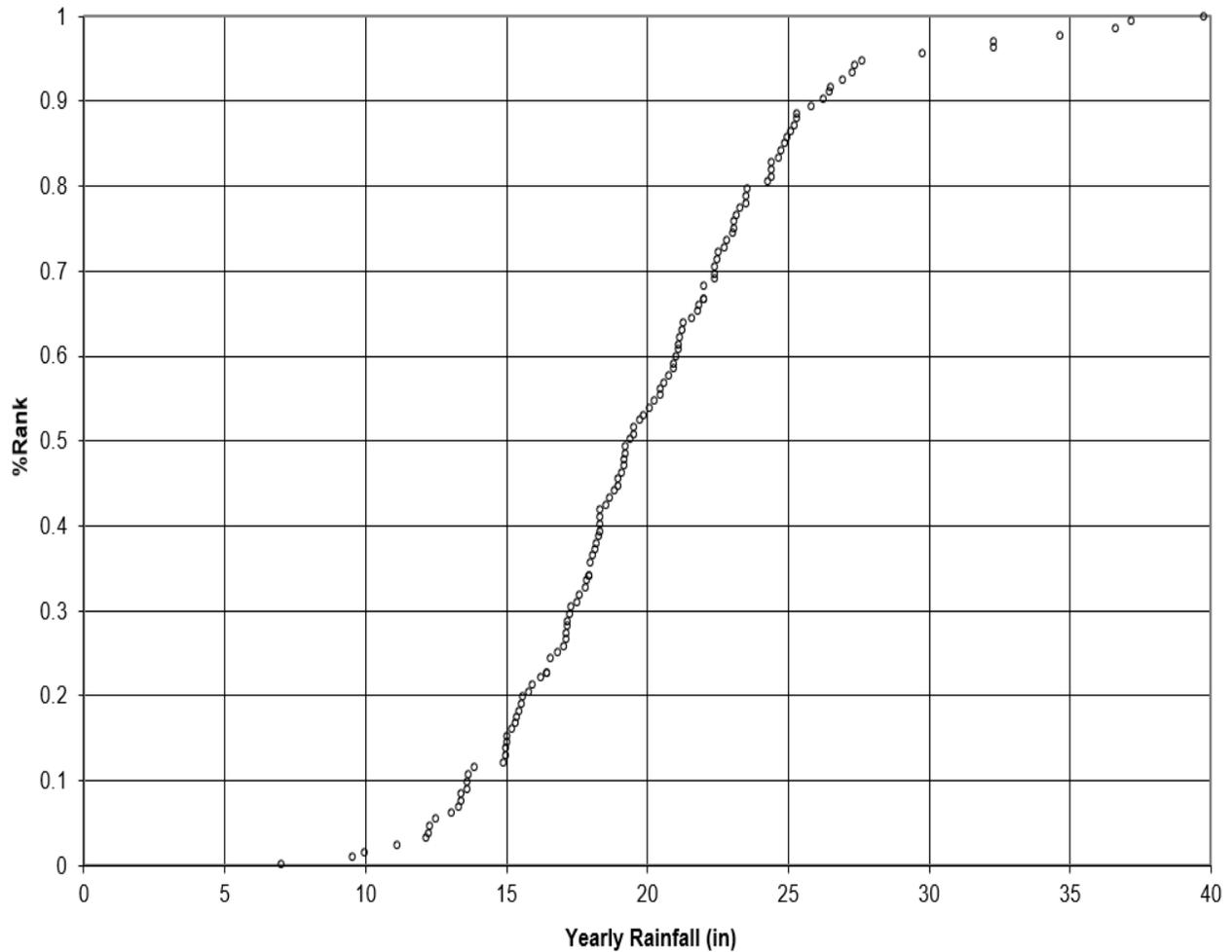
Amarillo's rainfall for 2024 was 20.95 inches. Seventy seven of the 133 years on record have been drier.

The purposes of this study were to (1) generate a cumulative distribution for the rainfall totals, (2) determine the nature of the distribution, and (3) determine the distribution parameters that would provide the best approximation to the data.

The analysis began by generating the cumulative distribution for the data. To accomplish this, the data were sorted in ascending order. Then a quantile was computed for each data value using Excel's PERCENTRANK.INC function.

A plot was then made with the % Ranks on the y-axis and the sorted rainfall totals on the x-axis, as shown in the following graph.

Amarillo's Yearly Rainfall Totals (1892 thru 2024)

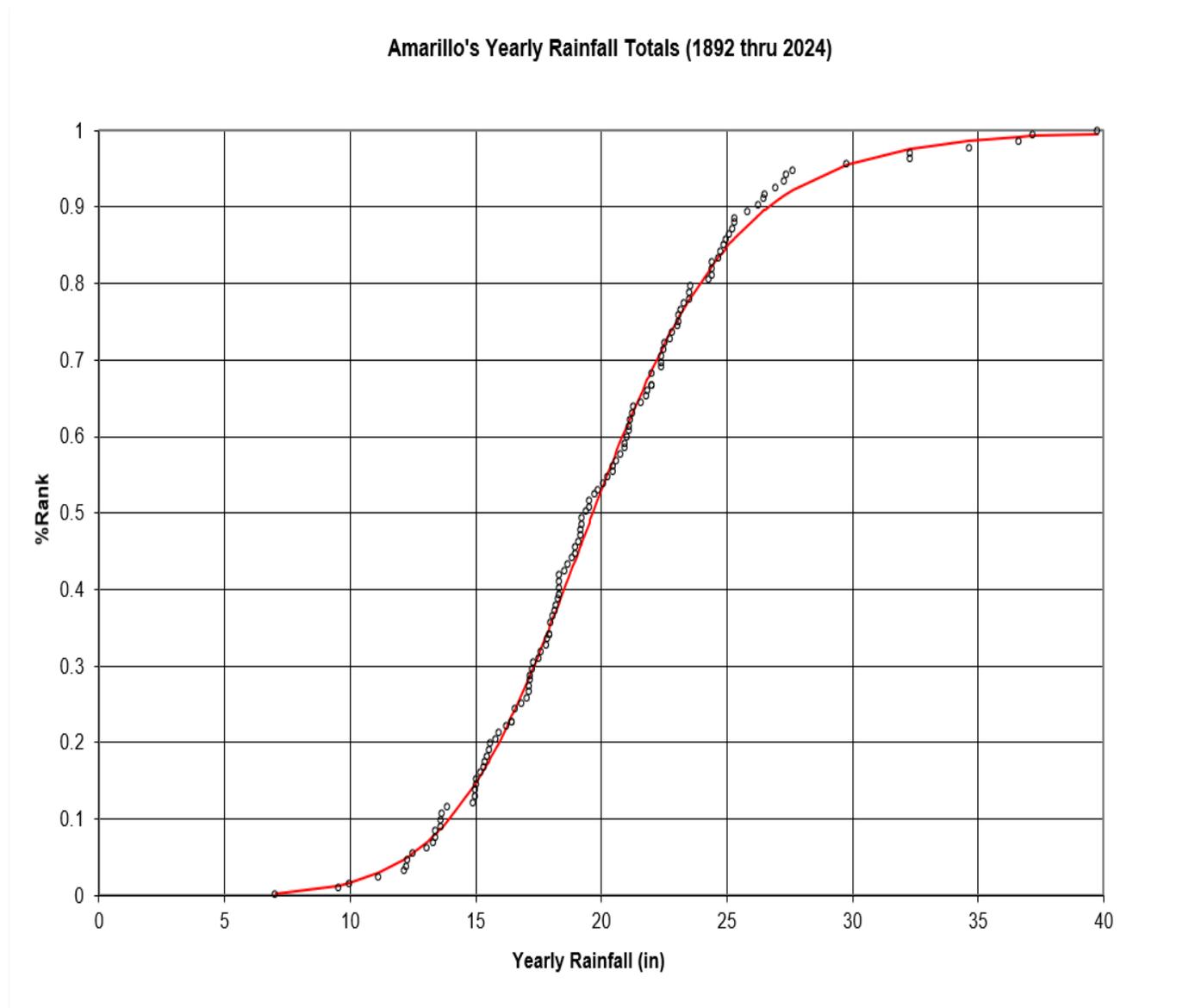


The data appear to be roughly “S” shaped, but are not symmetric since the right tail is longer than the left tail. Thus, we can rule out a normal distribution. Several asymmetric distributions were considered, such as log normal, square root normal, etc., but the distribution that fit the data the best was a modified sigmoid curve. Its equation is

$$F(x) = \frac{1}{1 + \exp\left(-\frac{x^\gamma - \alpha}{\beta}\right)}$$

where  $F(x)$  is the fraction of yearly rainfall totals that will be less than or equal to  $x$  inches, and  $\alpha$ ,  $\beta$ , and  $\gamma$  are parameters to be determined by a least squares regression.

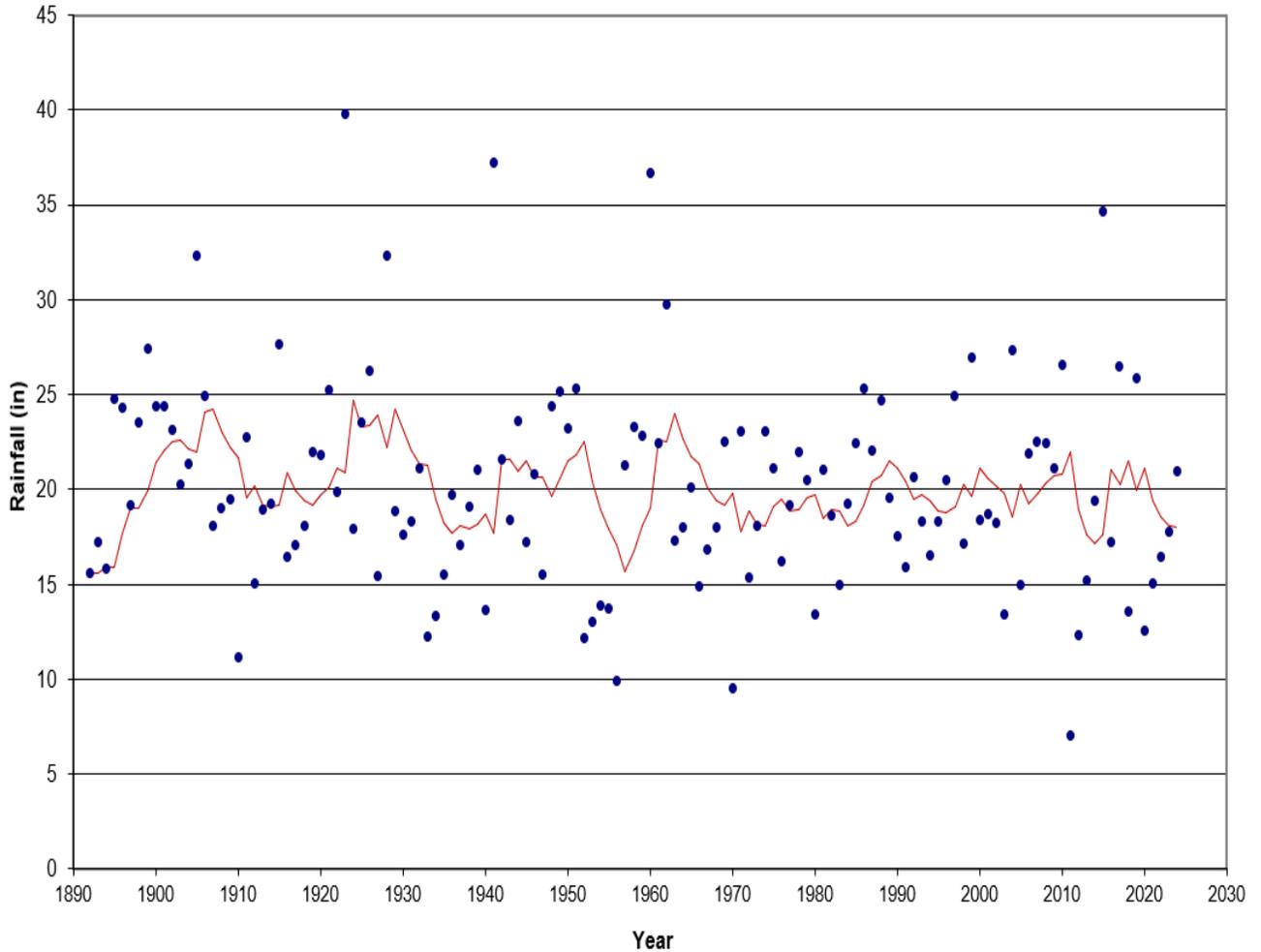
The parameters  $\alpha$ ,  $\beta$ , and  $\gamma$  were found to be 2.97093, 0.15910, and 0.36556, respectively. The resulting curve is shown with the data in the following plot, and the curve is an excellent approximation to the data.



Finally, here is a time series plot of the yearly rainfall totals (blue circles) with exponentially smoothed values (red line segments). The smoothed values indicate no apparent trend in the data.

## Amarillo's Yearly Rainfall Totals (1892 to 2024)

With Exponential Smoothing (Alpha = 0.2)



### Part 2. Monthly rainfall totals

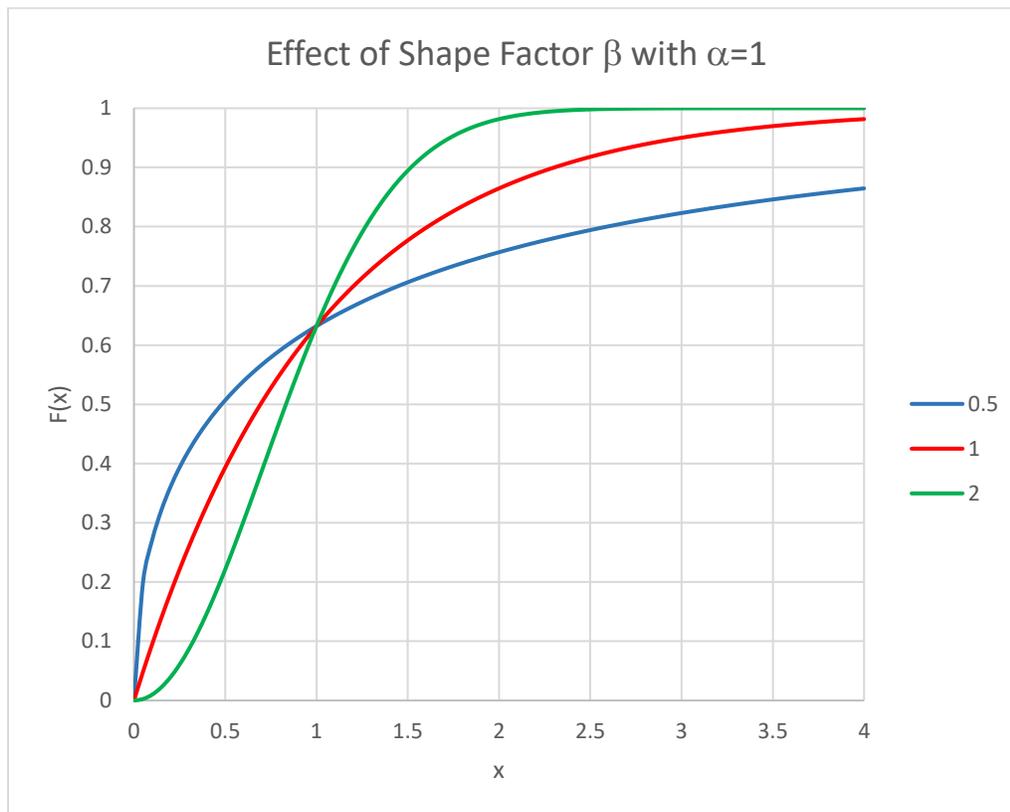
Amarillo's monthly rainfall totals required a different model than the one used for the annual totals. The monthly totals were found to be well approximated by a cumulative Weibull distribution function of the form

$$F(z) = 1 - e^{-z} \quad \text{where } z = \left(\frac{x}{\alpha}\right)^\beta$$

In this equation,  $F(z)$  is the quantile,  $x$  is the rainfall total in inches,  $\alpha$  is the scale factor, and  $\beta$  is the shape factor. More information about the Weibull distribution can be found at this website: [https://en.wikipedia.org/wiki/Weibull\\_distribution](https://en.wikipedia.org/wiki/Weibull_distribution)

As before, the rainfall totals for each month were sorted in ascending order and became the  $x$ -coordinates. A quantile  $q_i$  was calculated for each of  $n$  rainfall totals and the quantiles became the  $y$ -coordinates. The parameters  $\alpha$  and  $\beta$  for each month were found by least squares regression.

The following plot shows the effect of the shape factor  $\beta$ . For each curve,  $\alpha = 1$  while  $\beta = 0.5, 1, \text{ and } 2$ . For  $\beta > 1$ , the curve has an S shape. The curve is a smooth exponential for  $\beta \leq 1$ .

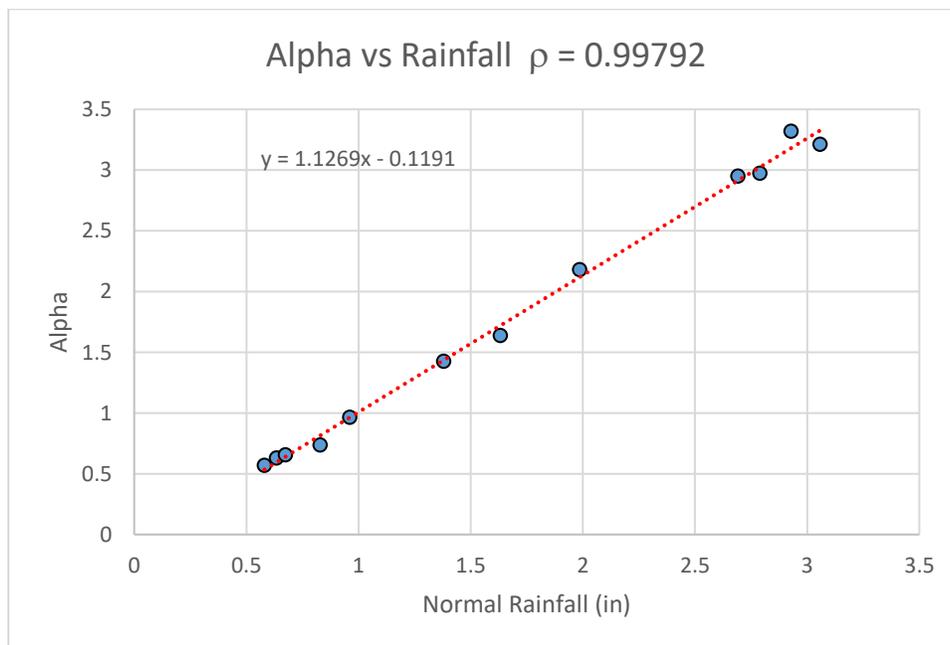


Least squares regression with the cumulative Weibull distribution function was done for each of the 12 months. A plot for each month is shown in the Appendix. Results of the curve fits are shown in the following table:

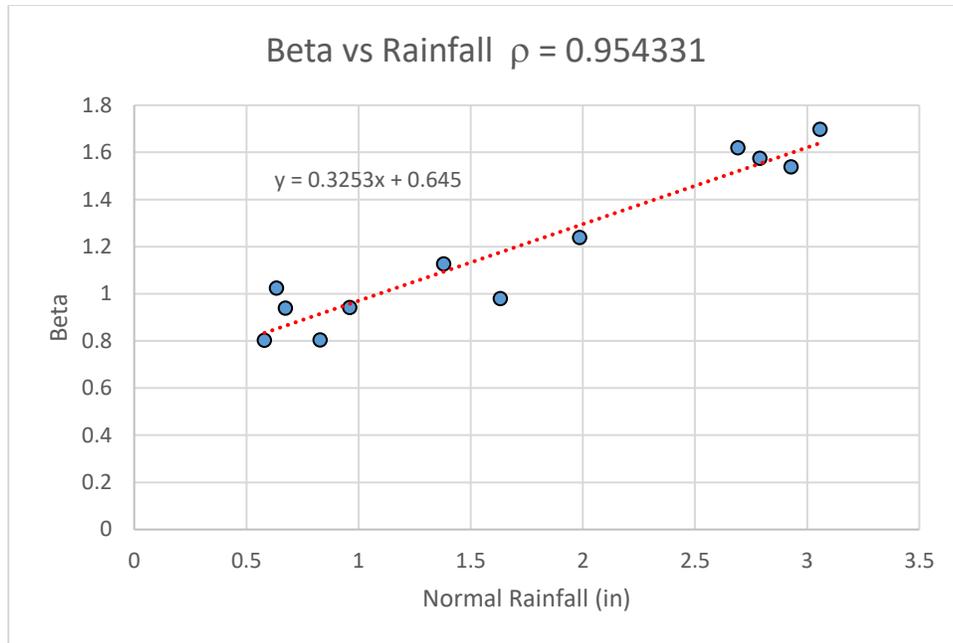
Month	Normal Rainfall (in)	Minimum (in)	Maximum (in)	$\alpha$	$\beta$	$R^2$
Jan	0.58015	0	3.17	0.569504	0.802262	0.994996
Feb	0.634211	0	2.93	0.629372	1.024589	0.99626
Mar	0.960827	0	4.14	0.966189	0.942259	0.99657
Apr	1.379023	0	0	1.425764	1.127337	0.997726
May	2.789323	0.04	9.81	2.975151	1.574928	0.996028
Jun	3.056767	0.01	10.73	3.211809	1.698429	0.996108
Jul	2.690526	0.04	8.02	2.9506	1.619564	0.995407
Aug	2.927669	0.15	8.07	3.319018	1.537879	0.997401
Sep	1.984962	0.03	6.42	2.181927	1.239462	0.998062
Oct	1.631729	0	7.64	1.63902	0.980143	0.994337
Nov	0.828195	0	6	0.73719	0.804458	0.991263
Dec	0.674436	0	4.52	0.65788	0.93867	0.996772

The table shows, for each month, the normal rainfall for that month, the minimum and maximum observed values,  $\alpha$  and  $\beta$  from the curve fit, and  $R^2$ , a measure of how well the curve fits the data. An  $R^2$  value of 1.0 indicates a perfect fit.

The scale factor  $\alpha$  is a good approximation to the normal rainfall. Note how the normal rainfall totals for each month are close to the corresponding  $\alpha$  values. These results suggest that there is a good correlation between the values of  $\alpha$  and the normal rainfalls. Here is the plot with the trendline:



Hence, the relationship is highly linear with a correlation coefficient of 0.99792. What about the shape factor  $\beta$ ? Does it also correlate with normal rainfall? Here is the plot with the trendline:



There is a tendency for  $\beta$  to increase with rainfall, but the effect is not as linear as it is with  $\alpha$ . The correlation coefficient has dropped to 0.954331.

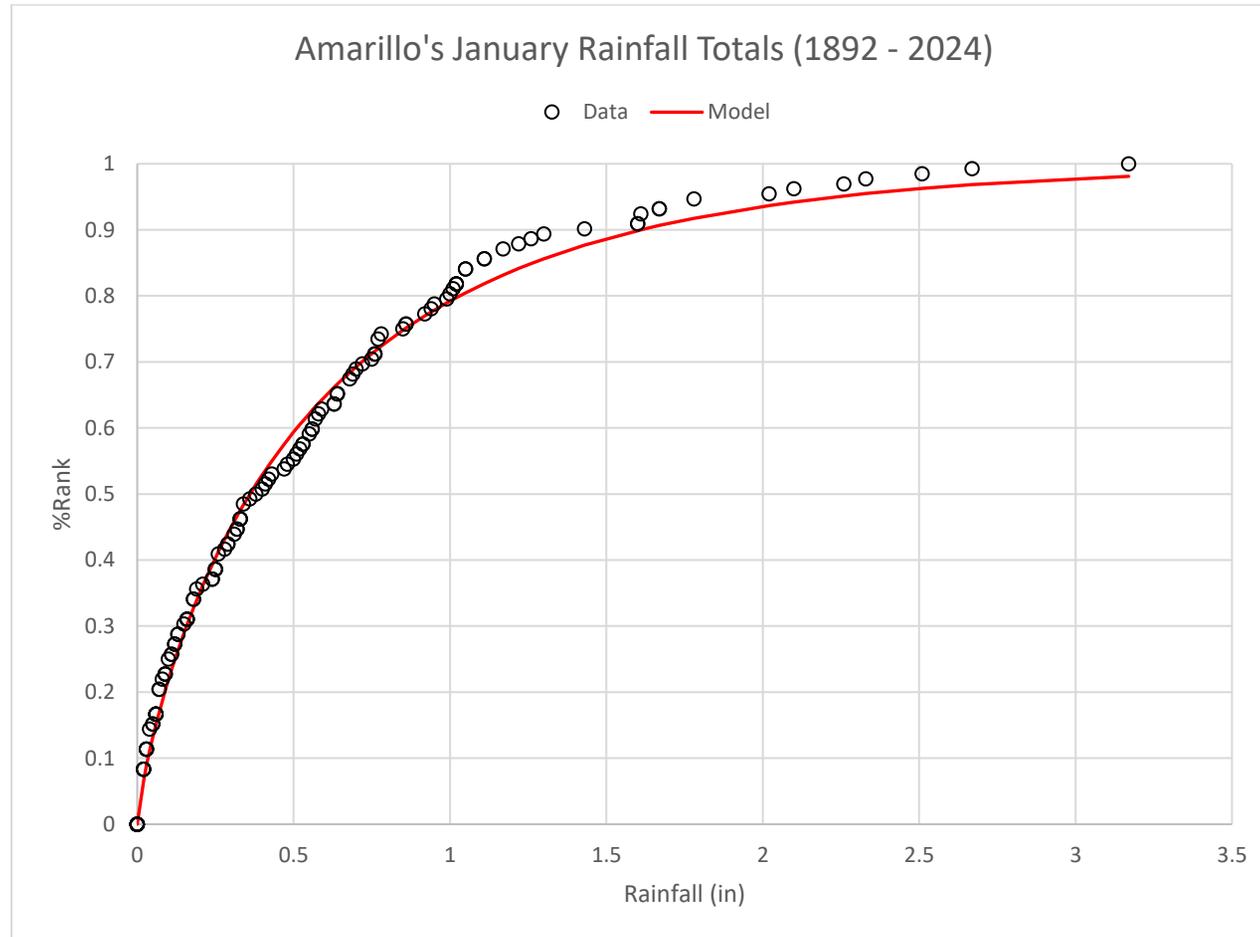
The best fit lines for  $\alpha$  and  $\beta$  as functions of normal rainfall ( $x$ ) are:

$$\alpha = 1.1269x - 0.1191$$

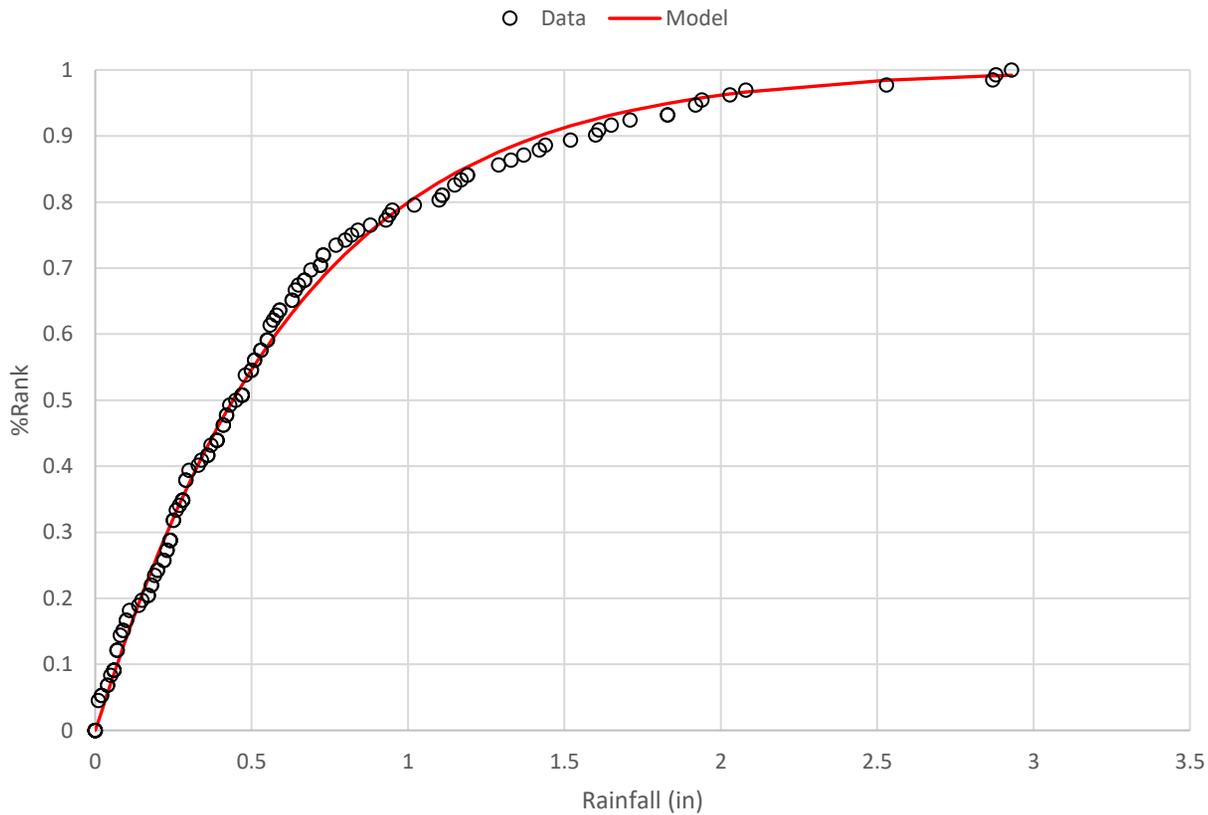
$$\beta = 0.3253x + 0.6450$$

In conclusion, Amarillo's monthly rainfall totals from the past 133 years can be approximated by cumulative Weibull distribution functions with parameters  $\alpha$  and  $\beta$ . The parameters have been shown to correlate with monthly normal rainfalls. Both parameters tend to increase as monthly rainfall totals increase.

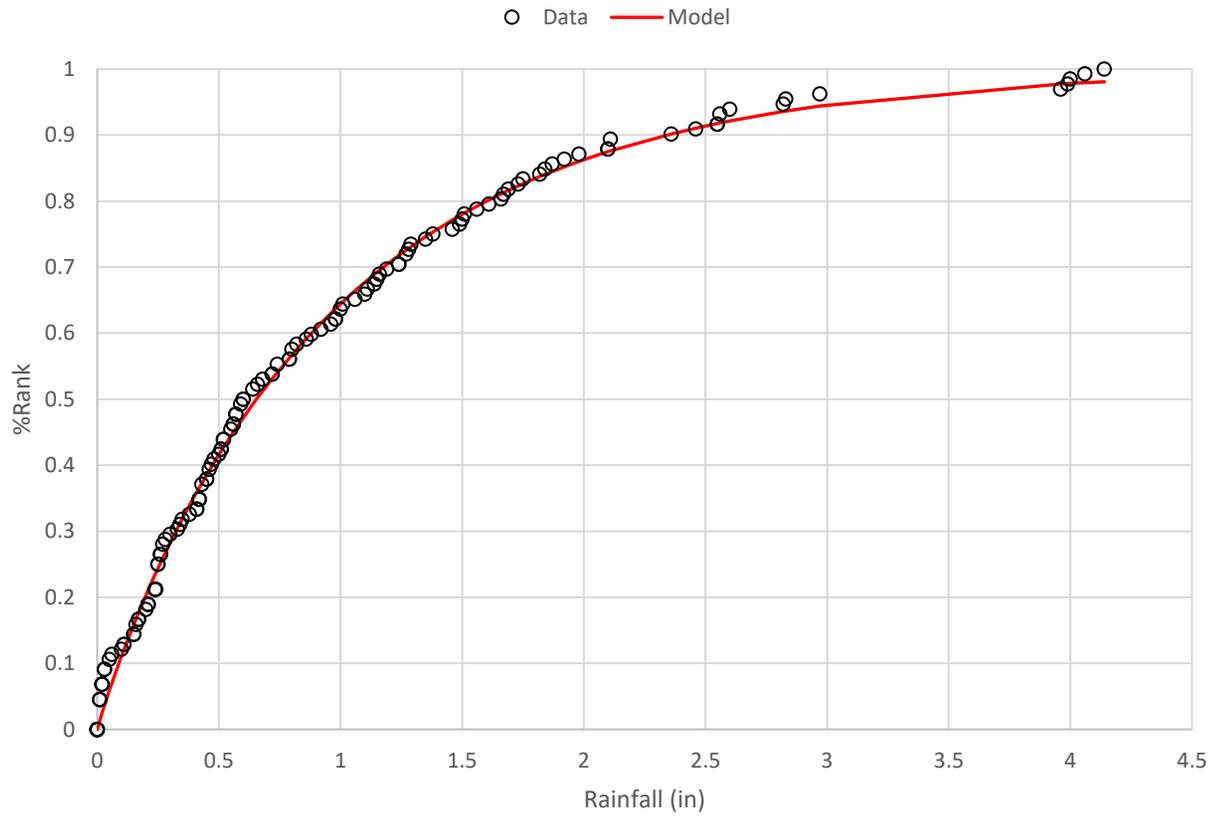
## APPENDIX



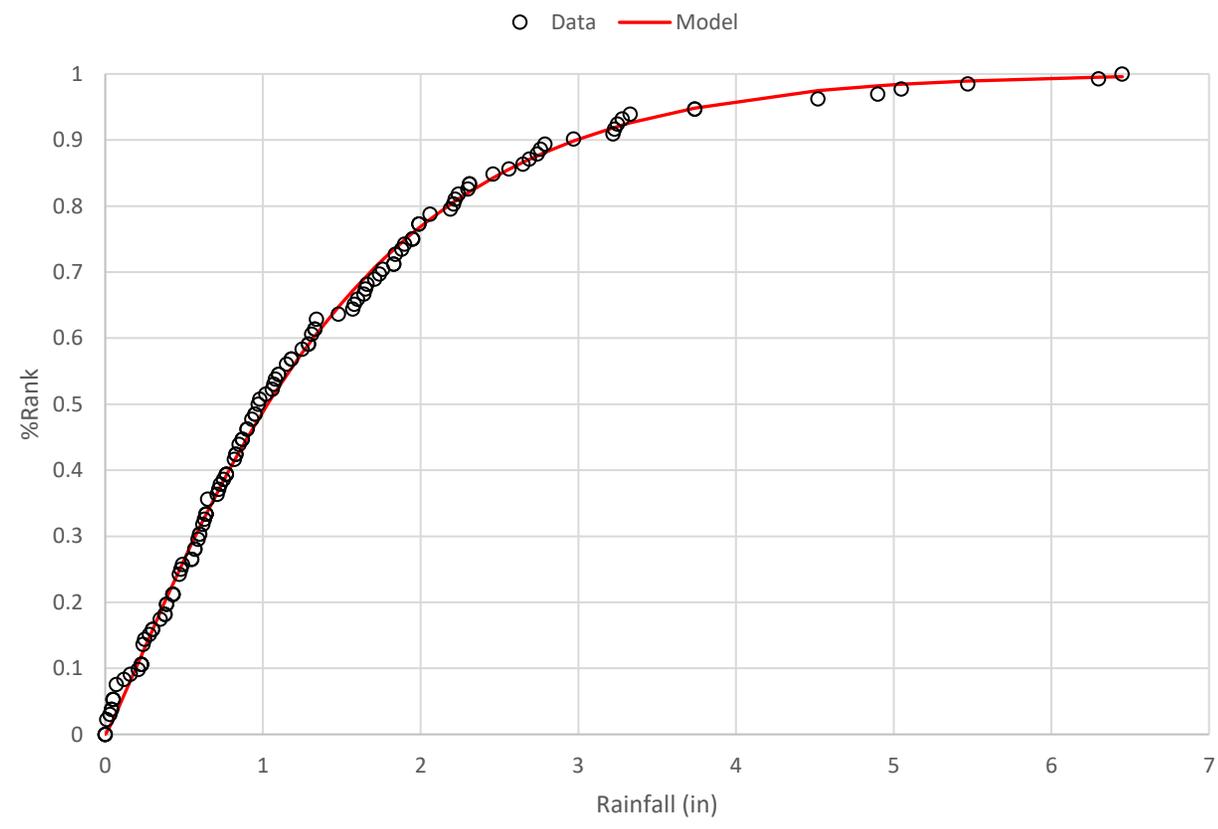
### Amarillo's February Rainfall Totals (1892 - 2024)



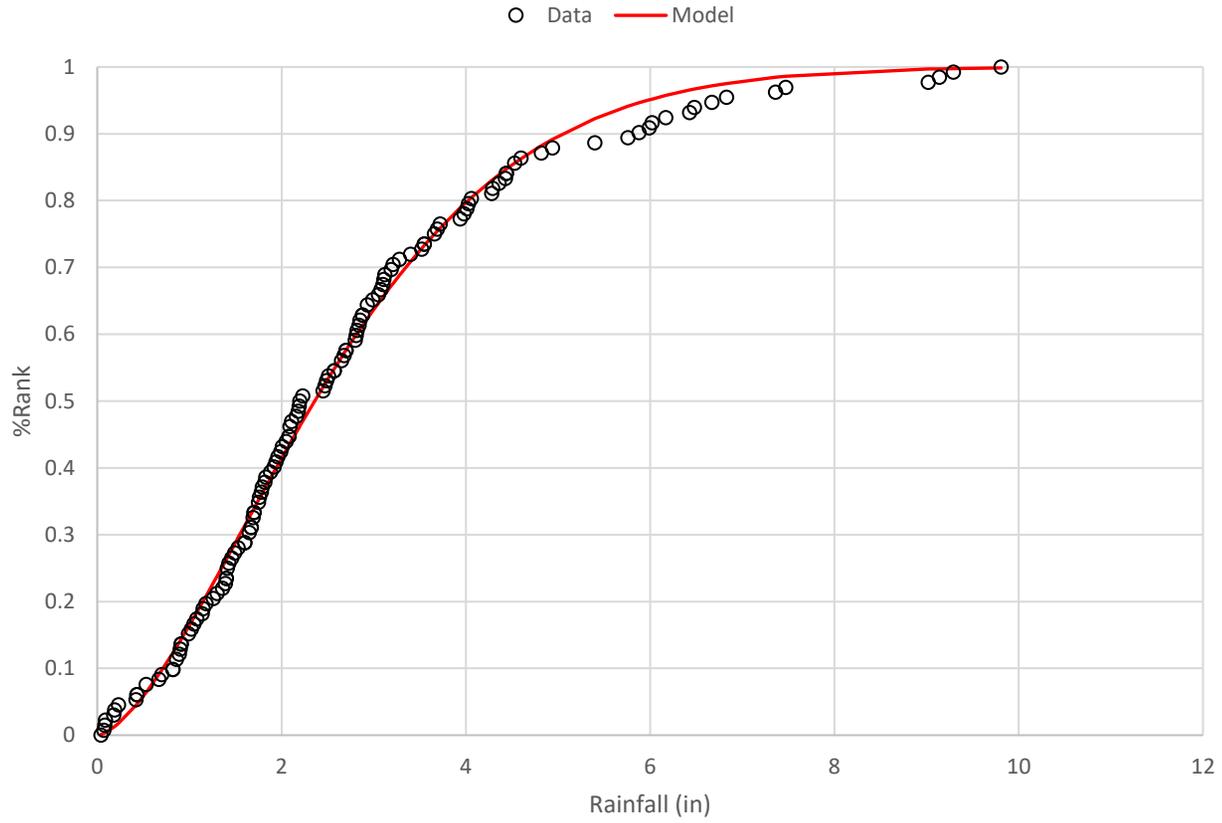
Amarillo's March Rainfall Totals (1892 - 2024)



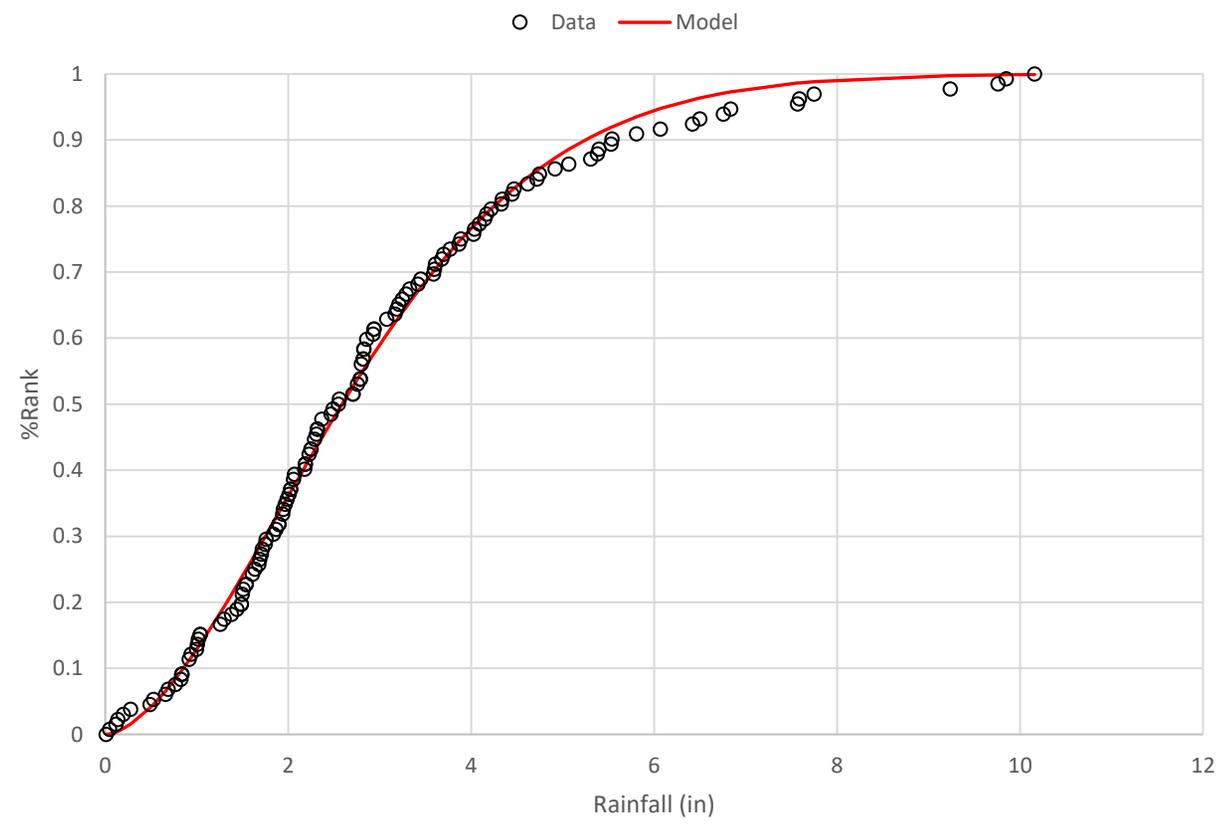
Amarillo's April Rainfall Totals (1892 - 2024)



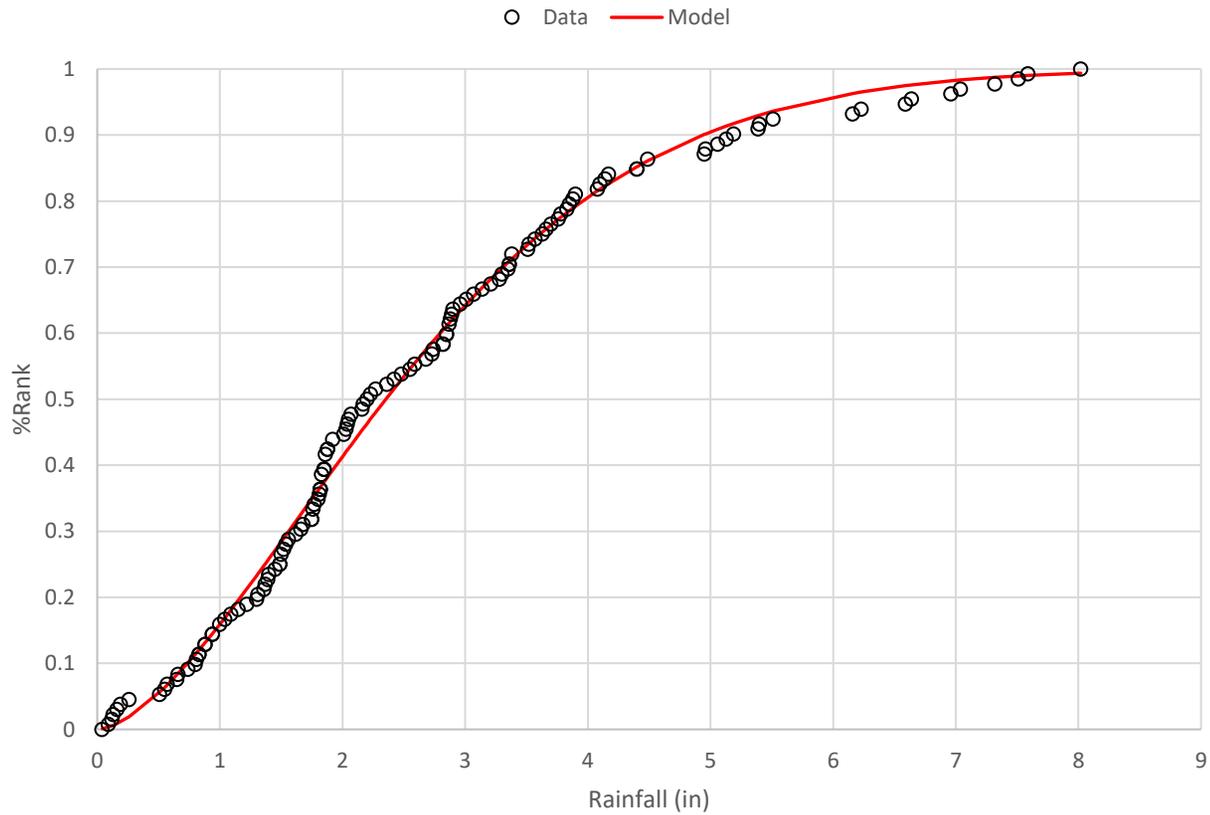
### Amarillo's May Rainfall Totals (1892 - 2024)



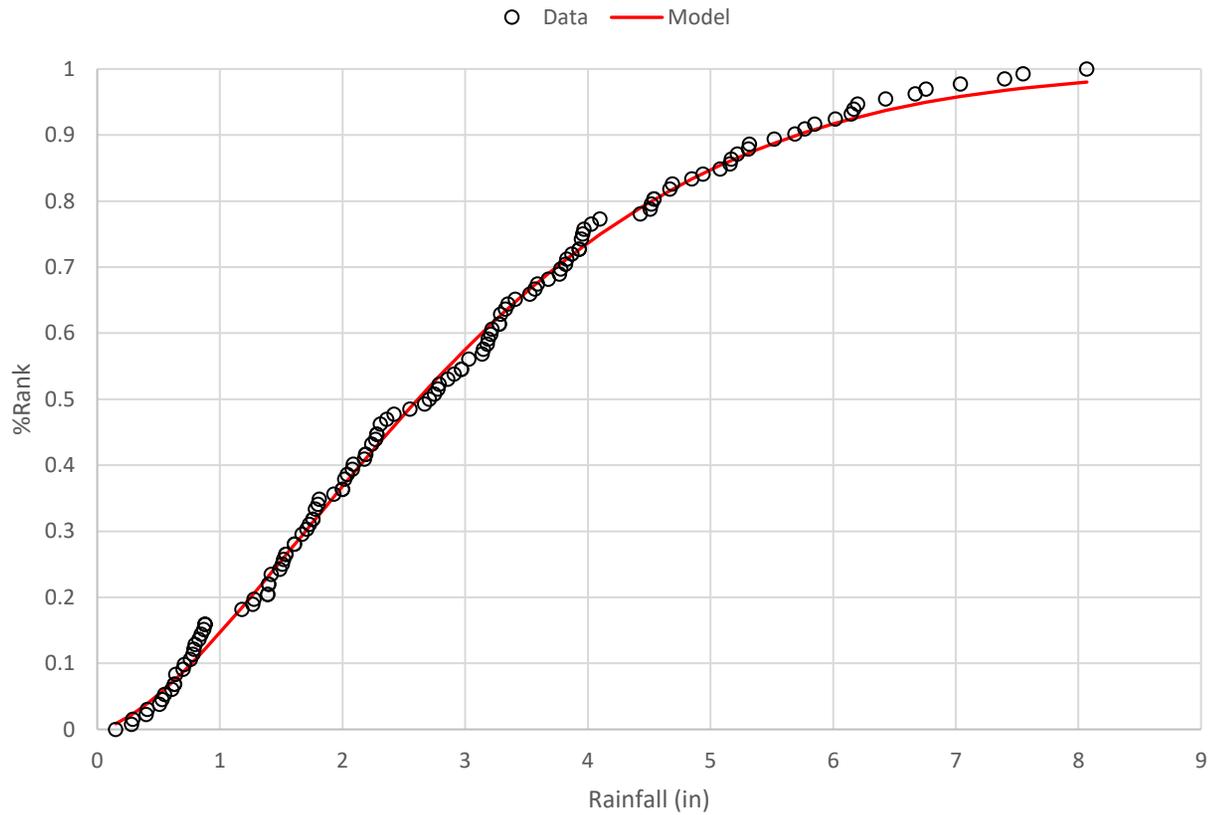
Amarillo's June Rainfall Totals (1892 - 2024)



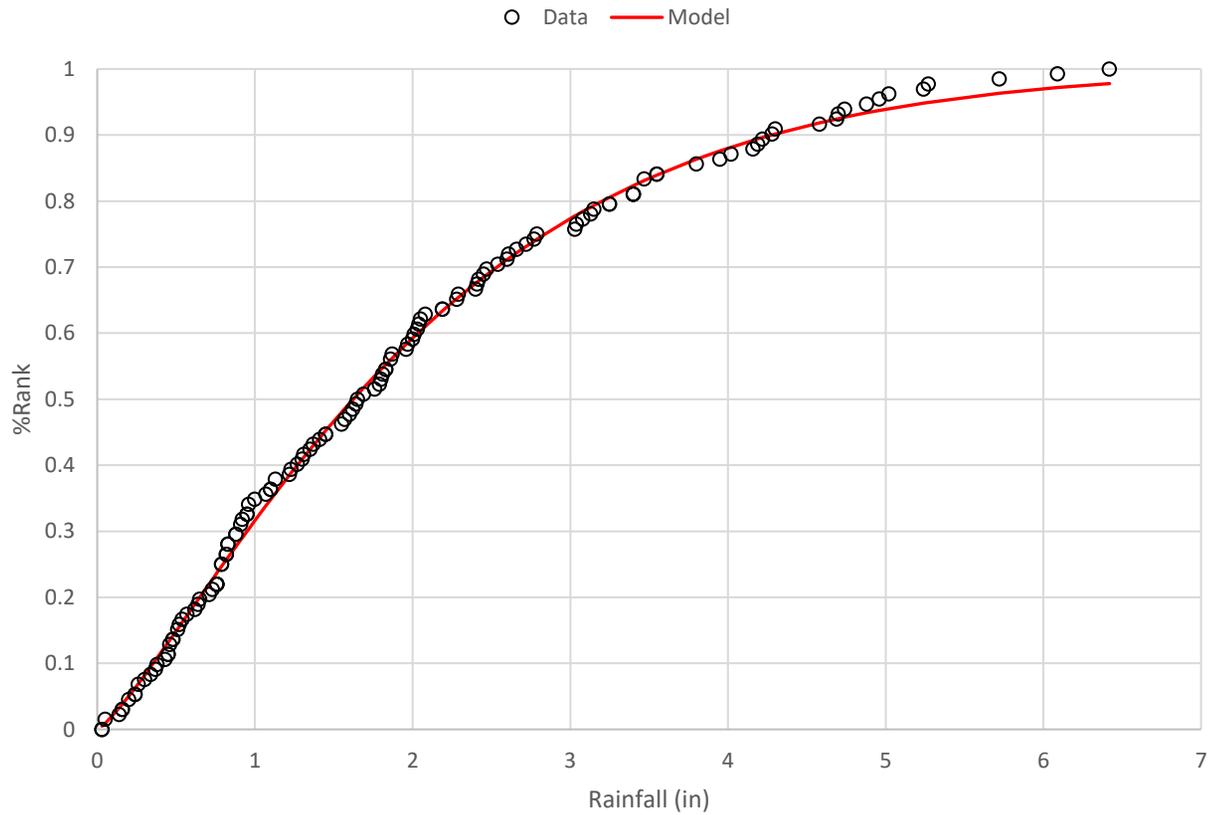
Amarillo's July Rainfall Totals (1892 - 2024)



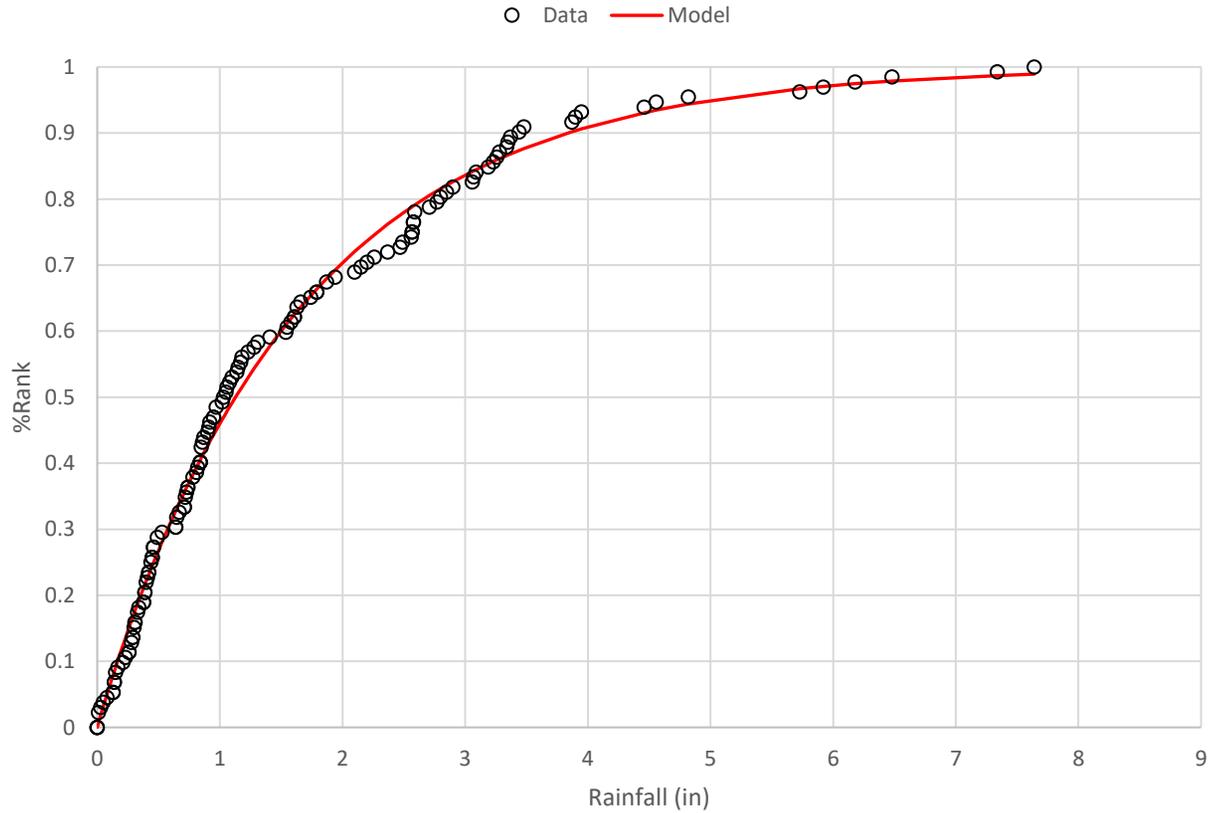
Amarillo's August Rainfall Totals (1892 - 2024)



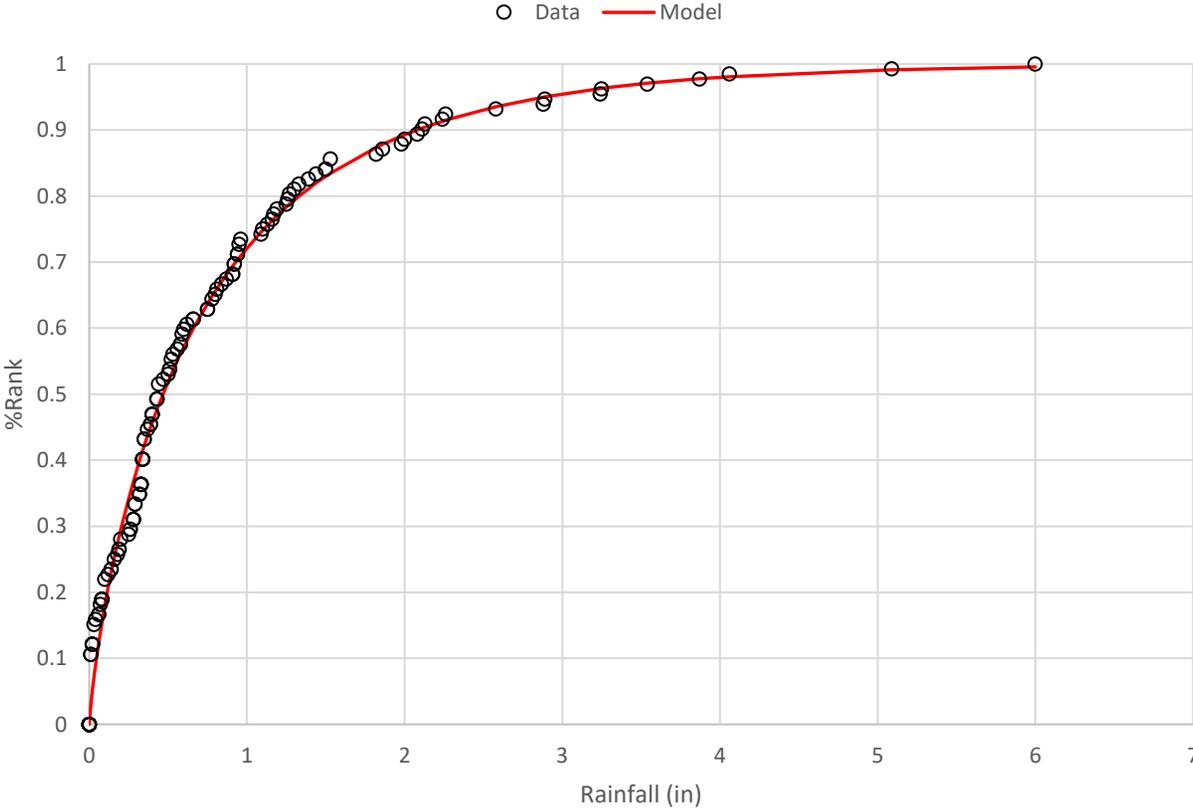
Amarillo's September Rainfall Totals (1892 - 2024)



Amarillo's October Rainfall Totals (1892 - 2024)



Amarillo's November Rainfall Totals (1892 - 2024)



Amarillo's December Rainfall Totals (1892 - 2024)

