Decaying Average BMA

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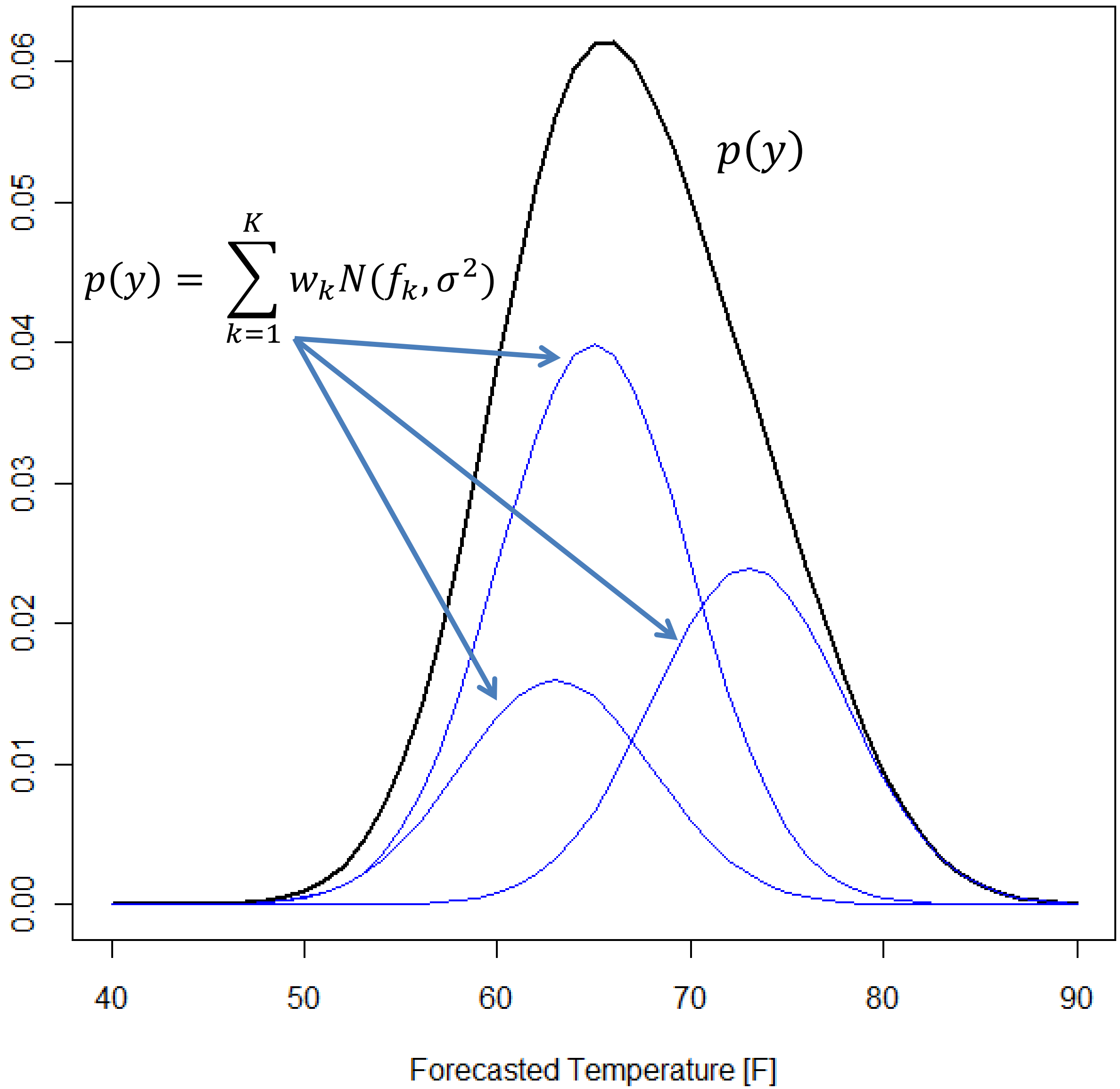
Introduction

- Raftery et al. (2005) proposed applying Bayesian Model Averaging (BMA) to ensembles
- Basic Idea
  - Weight ensemble members based on past performance
  - Calibrate ensemble spread
  - Do this by fitting a Normal Mixture statistical model to ensemble member forecasts

Normal Mixture Model



Fitting the Statistical Model

- Challenge is to estimate statistical model parameters

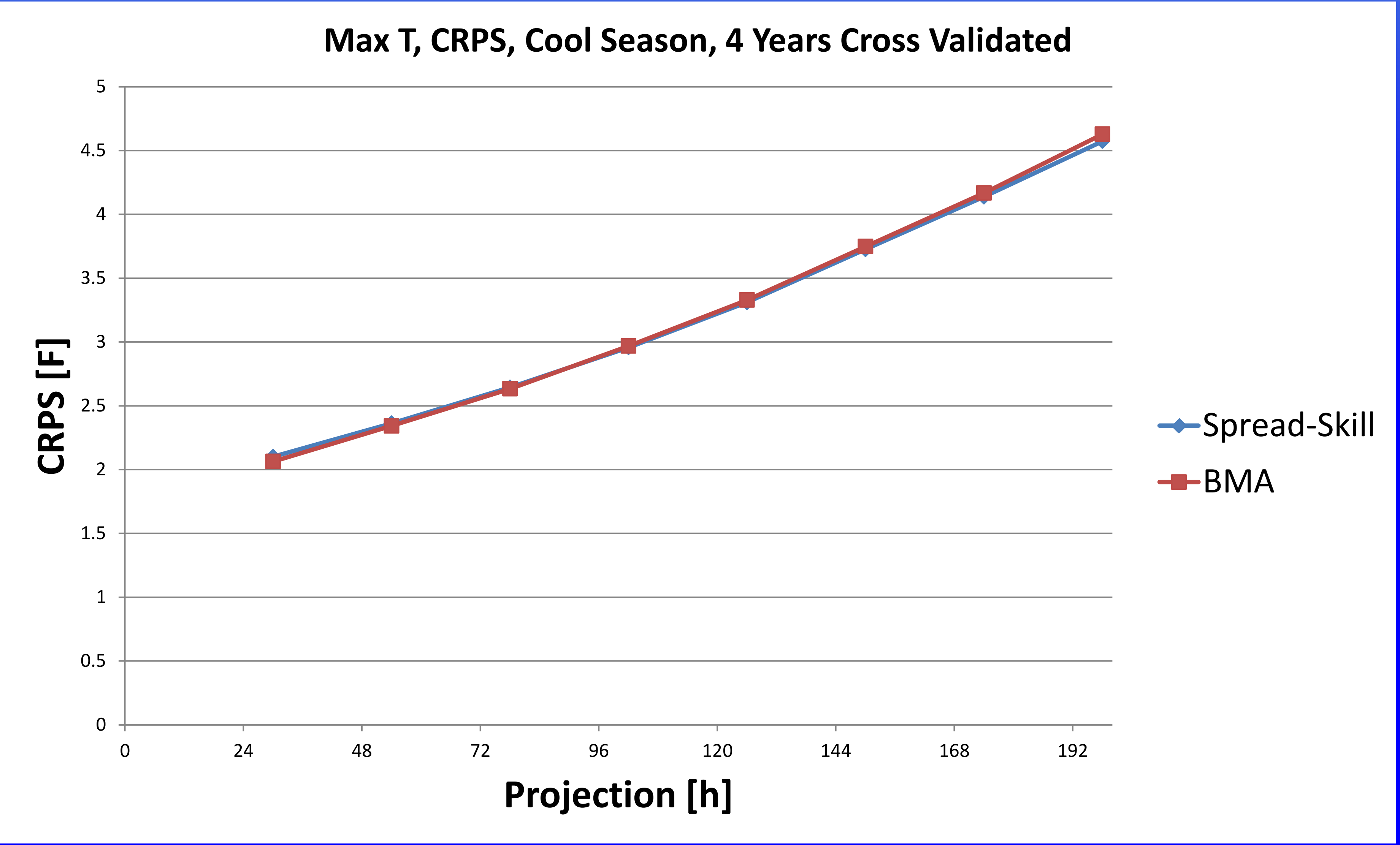
$$\boldsymbol{p(y) = \sum_{k=1}^{K} w_k N(f_K, \sigma^2)}$$

Where

w_k are weights -> $p(M_k|y^T)$
f_k is the ensemble-member forecast
σ is the predictive variance

- Raftery et al. (2005) estimated parameters with the Expectation Maximization Algorithm (Dempster et al., 1977).

Example NAEFS Application

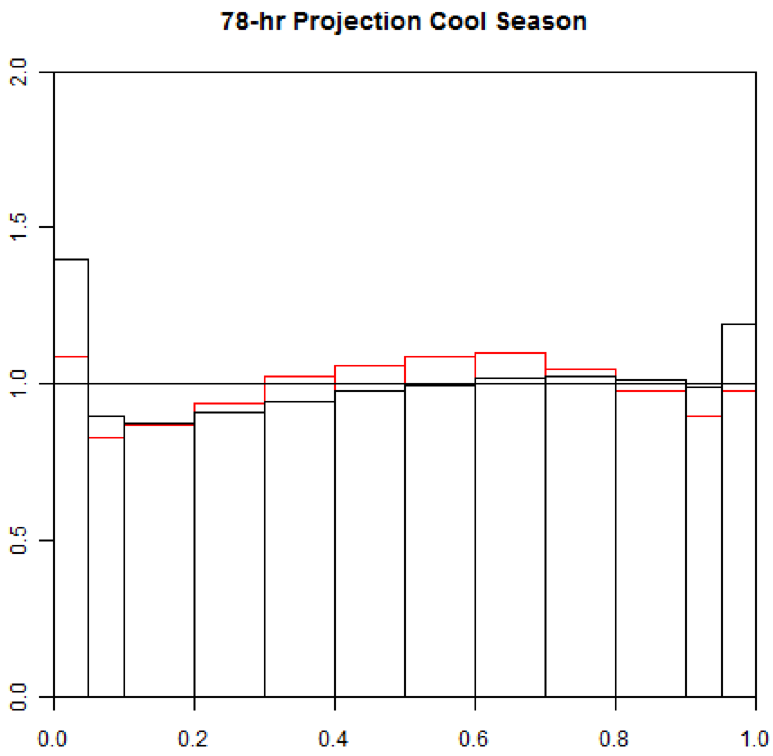


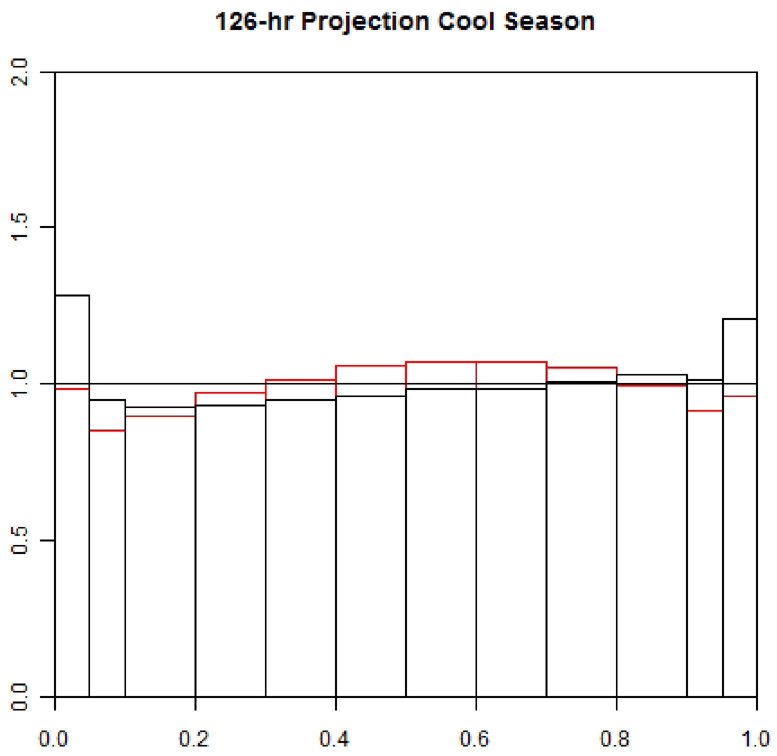
Example NAEFS Application

Spread-Skill

BMA

Max T, Cool Season, 4 Years, Cross Validated





Fitting the Statistical Model

- EM Algorithm
  - Iterative
  - Must keep entire training sample on hand
  - Prone to overfitting with small samples (Hamill 2007)
- Propose Decaying Average BMA
  - Estimate parameters with decaying averages rather than EM algorithm
  - Stable estimates
  - Less data storage
  - Results comparable to EM algorithm
  - Similar to NCEP's bias correction

Decaying Average BMA

- Use similar formulation to Raftery et al. (2005)
- Continuously update estimate of weights and predictive standard deviation as past forecasts verify
- Update is via a decaying average

$$\text{New Estimate} = (0.95 \times \text{Old Estimate}) + (0.05 \times \text{Latest Estimate})$$

Decaying Average BMA

- Issue a forecasts
  - – For example, the 42 hour 2-m temperature forecast
- Wait for forecast to verify
  - – Different projections verify at different times
- Pair forecast with its verifying observation
- Begin update process

Decaying Average BMA

- Two-step procedure
  - First update weights
  - Then update predictive standard error
- Going to demonstrate procedure for updating weights
  - Update for predictive error is similar

For one case, take member forecasts and observation, and compute…

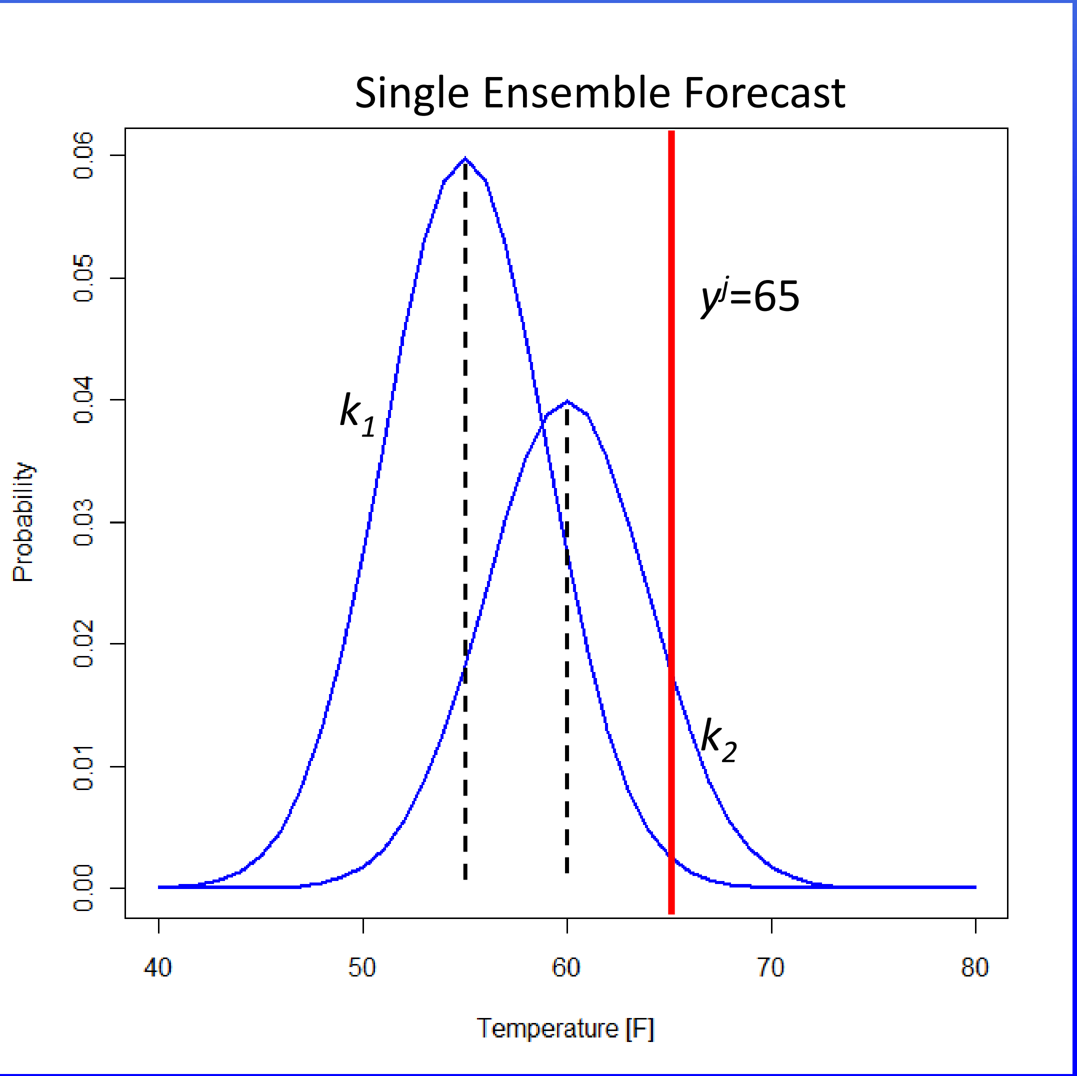
$$z_k^j = \frac{w_k^{j-1} g(y^j | f_k^j, \sigma^{j-1})}{\sum_{i=1}^{K} w_i^{j-1} g(y^j \,| f_k, \sigma^{j-1})}$$

w_k^{j-1}	Previous weight estimate for member *k*
$g(y_t | f_{kt}, \sigma^{j-1})$	$N(f_k^j, \sigma^{j-1})$ evaluated at observation y_t.
k	Ensemble Member
j	Current day being verified
y_j	Observation

Example Z calculation

$$z_k^j = \frac{w_k^{j-1} g(y^j | f_k^j, \sigma^{j-1})}{\sum_{i=1}^{K} w_i^{j-1} g(y^j \, | f_k, \sigma^{j-1})}$$

$$z_1^j = \frac{0.005}{0.005 + 0.020}$$



Example Z calculation

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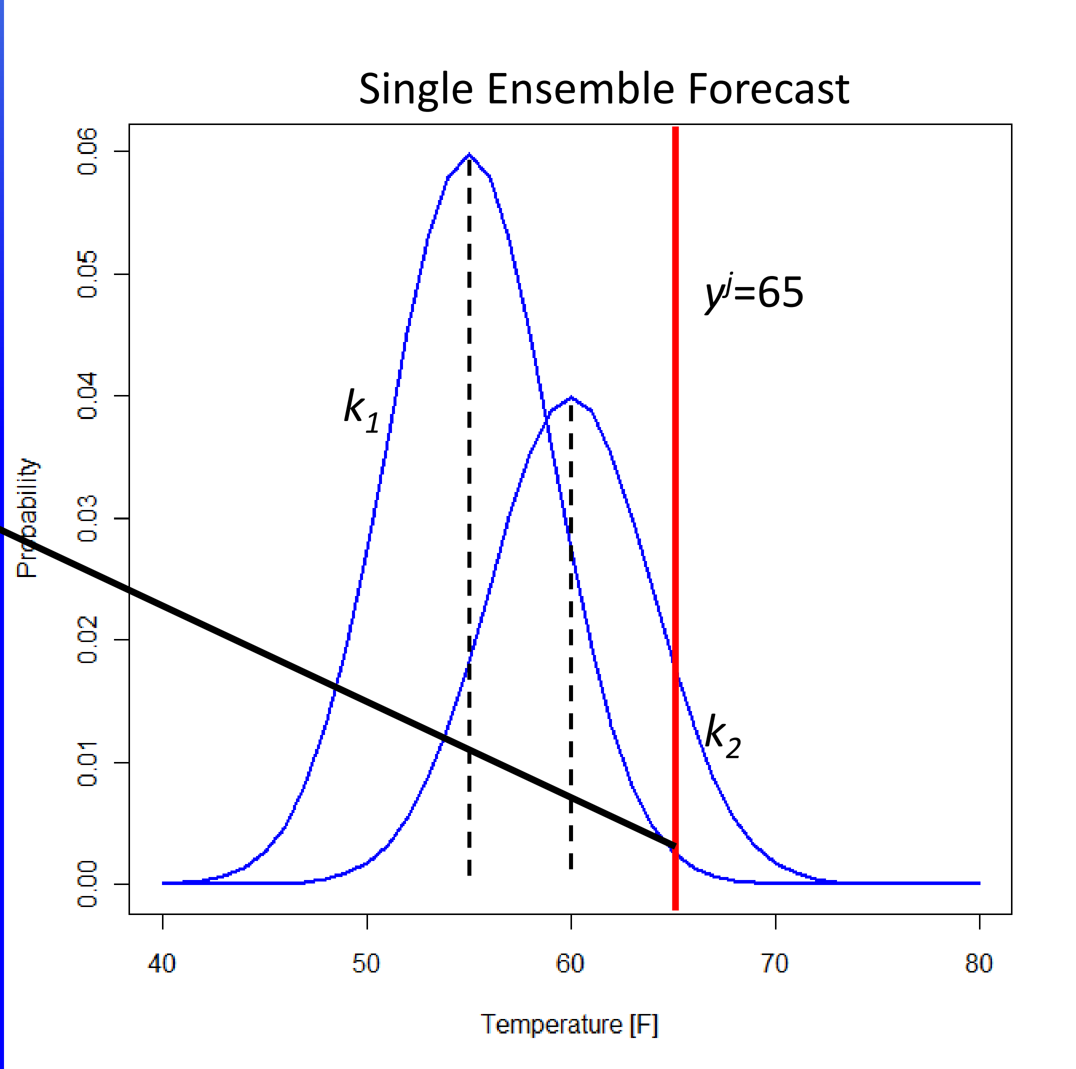
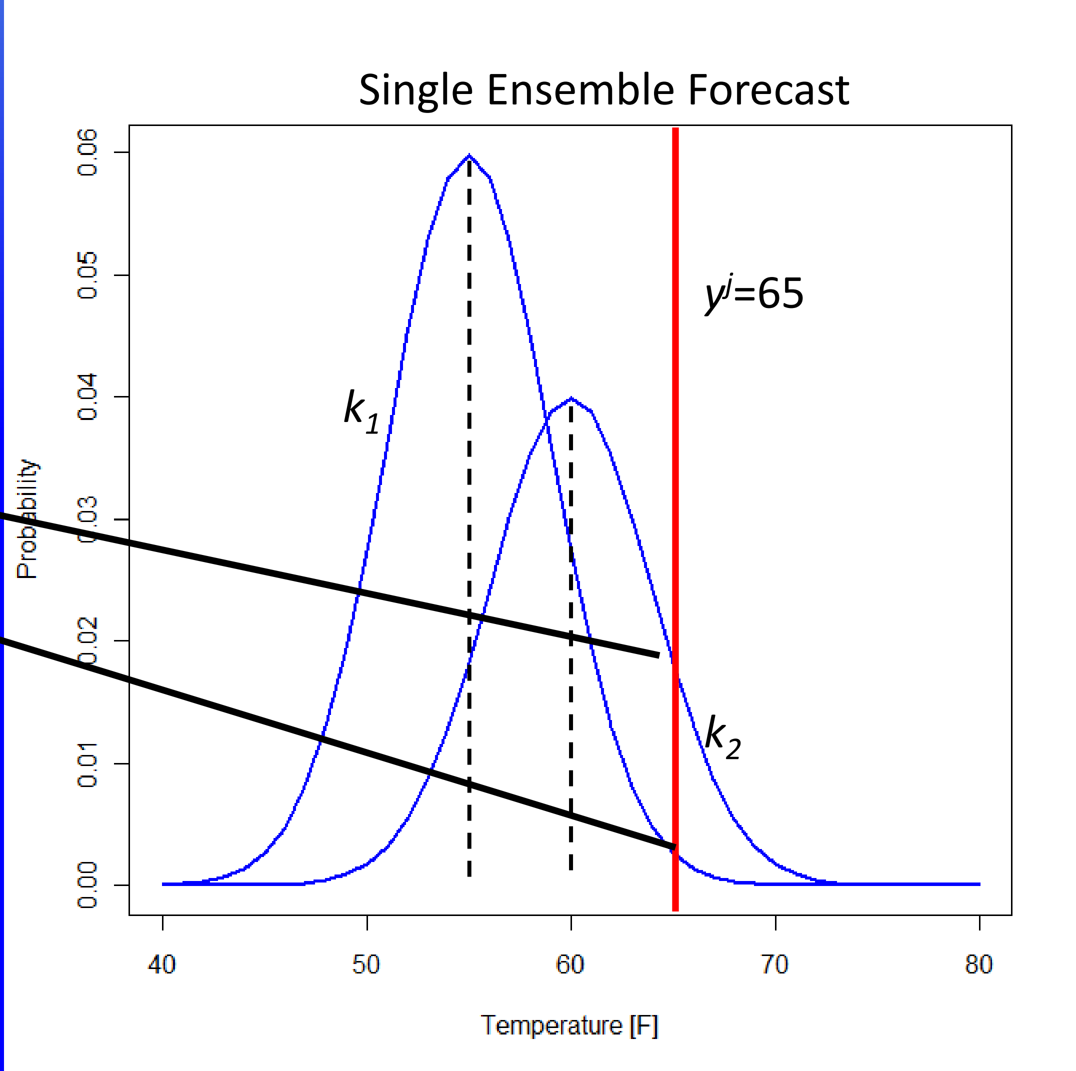


Example Z calculation

$$z_k^j = \frac{w_k^{j-1} g(y^j | f_k^j, \sigma^{j-1})}{\sum_{i=1}^{K} w_i^{j-1} g(y^j \,| f_k, \sigma^{j-1})}$$

$$z_1^j = \frac{0.005}{0.005 + 0.020}$$

$$z_1^j = 0.2$$



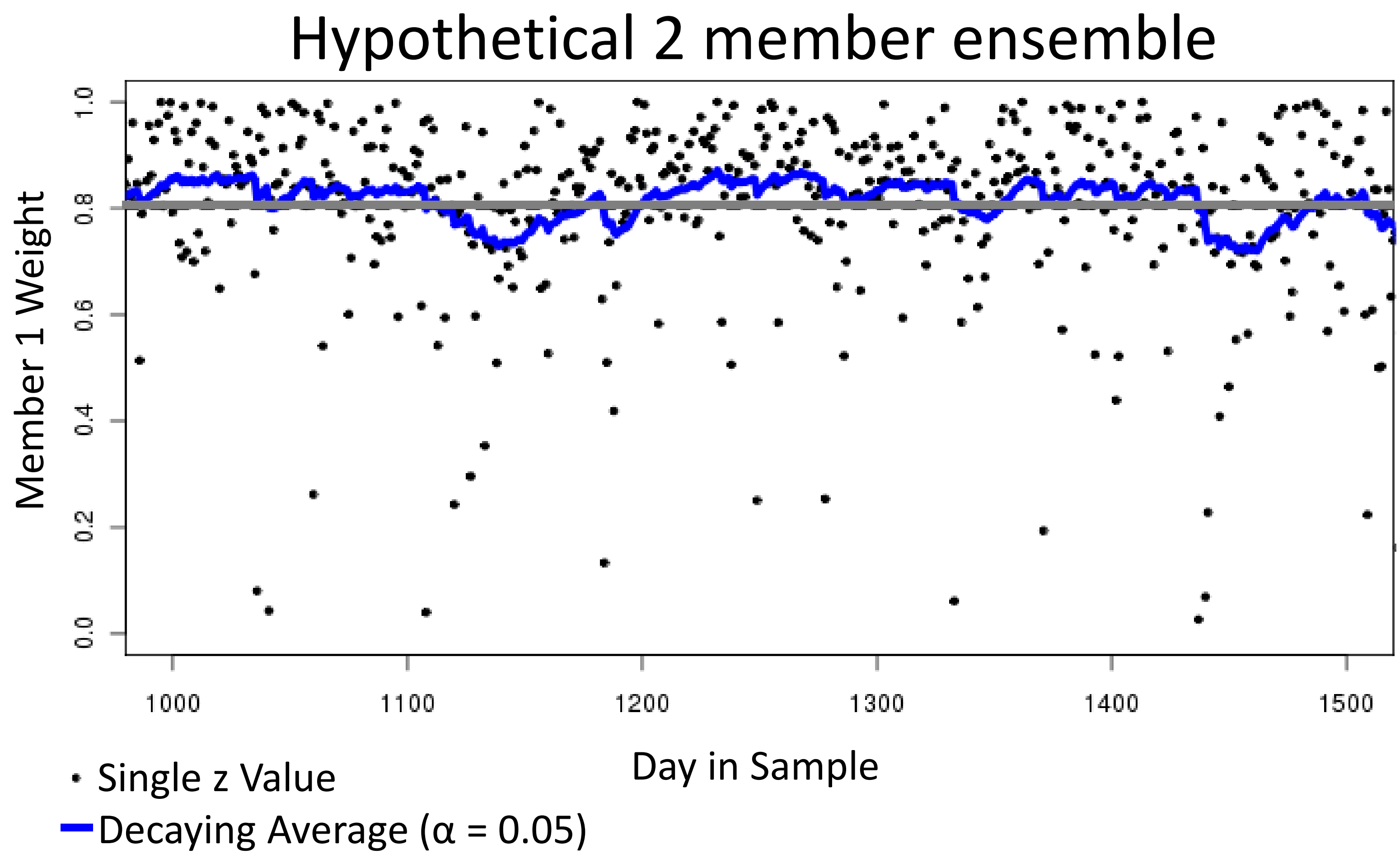
Updating the Weights

- $z^j{}_k$ values enter into decaying average update

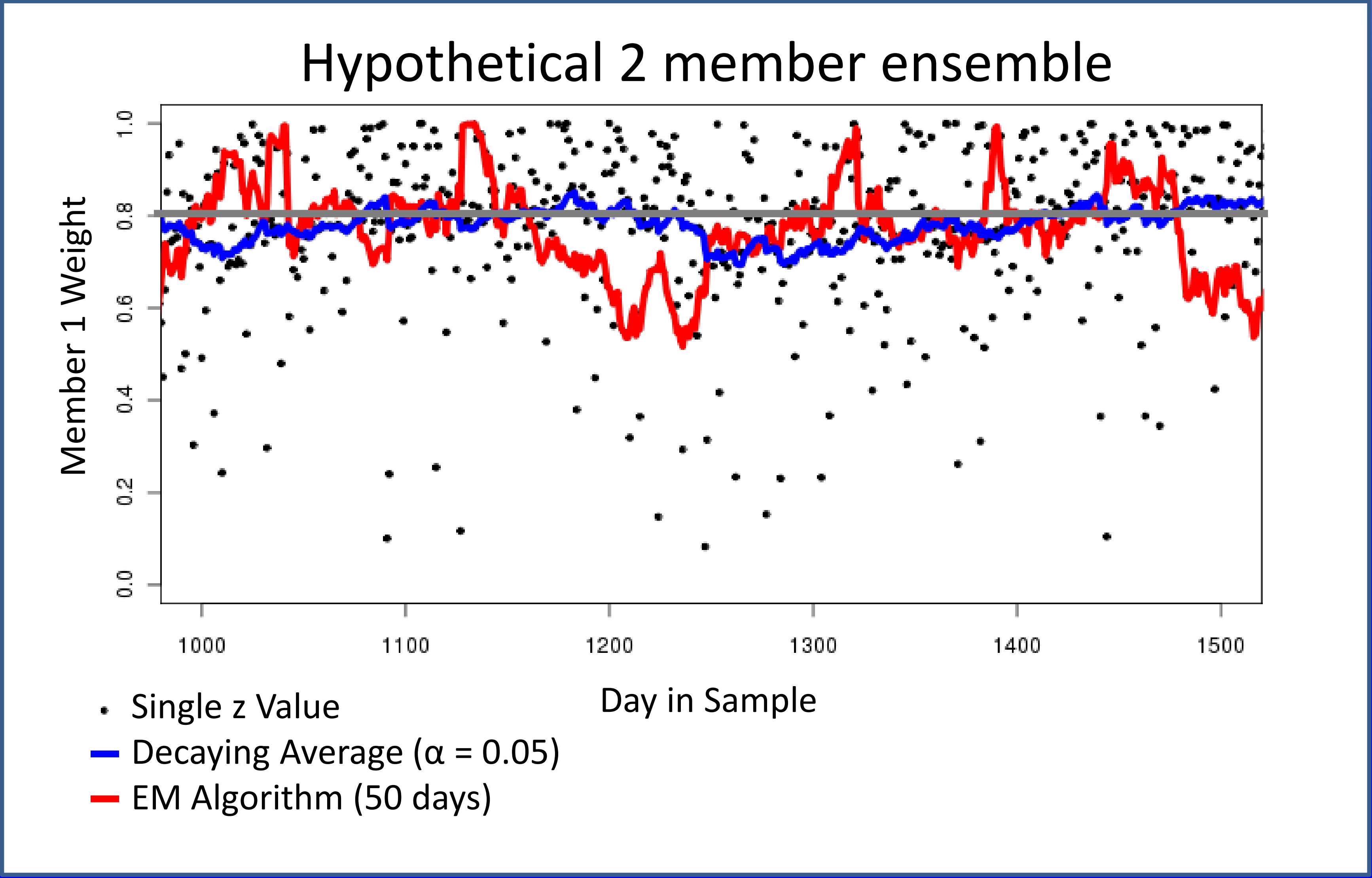
$$w_k^j = (1-\alpha) \times w_k^{j-1} + \alpha \times z_k^j$$

α Decaying weight (~.05)

Decaying Average BMA Example



Comparison with Raftery's BMA



Decaying Average BMA Spread

- With today's z^j_k values compute

$$s^{2(j)} = \sum_{k=1}^{K} z_k^j \left(y^i - f_k^j\right)^2$$

k	Ensemble Member
j	Current day being verified
y_j	Observation
f_j	Ensemble Member Forecast k

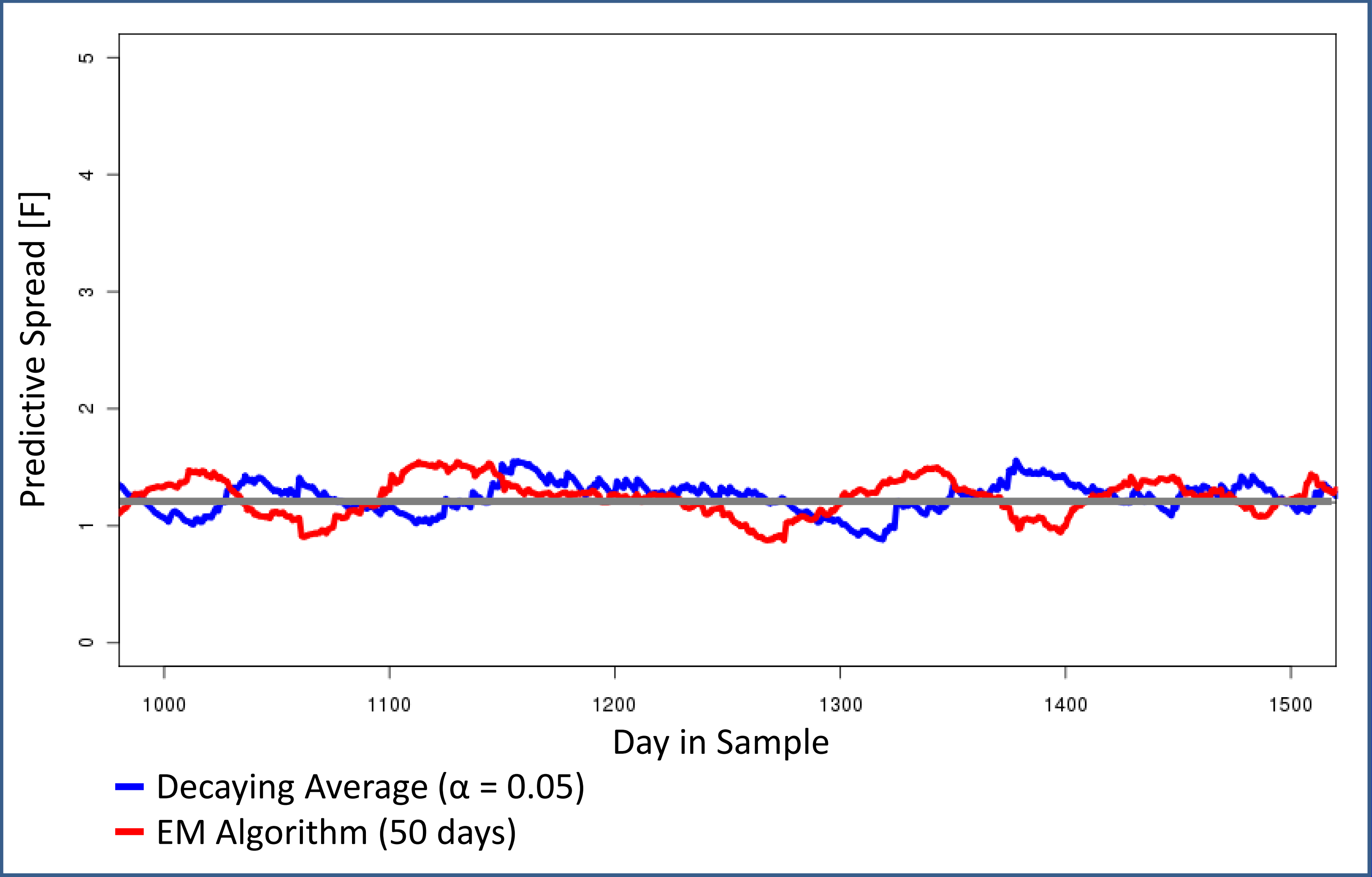
Decaying Average BMA Spread

- $s^{2(j)}$ enters into the decaying average algorithm

$$\sigma^{2^{(j)}} = (1 - \alpha) \times \sigma^{2^{(j-1)}} + \alpha \times s^{2^{(j)}}$$

α Decaying weight (~.05)

Comparison with Raftery's BMA



Conclusions

- Propose to use Decaying Average BMA
  - Stable parameter estimates
  - Less data storage (~3 days)
  - Avoids iterative algorithm
  - Results asymptotically similar to EM algorithm
- SREF
  - 21 members – 3 distinct models
  - 7 member sub-ensembles -> 3 weights, 1 standard deviation