FORECASTING THE CONDITIONAL PROBABILITY OF FROZEN PRECIPITATION

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1. INTRODUCTION

Information as to whether expected precipitation will be frozen or liquid is an important aspect of most weather forecasts issued to the public. Numerical weather prediction models do not directly produce this kind of information; however, they do contain variables which are strongly related to precipitation type. One such variable which has been used for many years--even before the days of numerical prediction--is 1000-500 mb thickness. Studies such as that of Wagner (1957) have produced valuable aids for the practicing forecaster. This paper describes the development of a system--and its use in the National Weather Service (NWS)--which produces objective forecasts of the conditional probability of frozen precipitation (PoFP(P)).

The technique used is called Model Output Statistics, or MOS. In this technique, the predictand is related statistically to variables which have been predicted by a numerical model or models. Therefore, the biases and other inaccuracies in the models are considered in the development of the relationships. Consideration of the model errors is particularly important when the forecasts are to be expressed in probability form. We feel that MOS is the most practical way of approaching the objective prediction of most weather elements. This is borne out by our success with forecasting probability of precipitation (Glahn and Lowry, 1969), conditional probability of frozen precipitation (Glahn and Lowry, 1972), surface wind (Glahn, 1970, and Barrientos, 1970), maximum temperature (Annett, et al., 1972), and ceiling and visibility (Bocchieri and Glahn, 1972).

Development of the PoFP(P) forecast system consisted of two basic steps. First, for each of 186 stations, we found a "50-percent" value for each of three variables predicted by the National Meteorological Center's Primitive Equation (PE) model (Shuman and Hovermale, 1968): 1000-500 mb thickness, boundary layer (B.L.) temperature, and 850-mb temperature. For instance, we found the value of the 1000-500 mb thickness which indicates a 50-50 chance of frozen precipitation at a particular station, provided precipitation occurs. These 50-percent values were determined by using the logit model to fit data from three winter seasons.

Secondly, the deviations from the 50-percent values were determined for each station for each variable; the relative frequency (for those cases when precipitation occurred) of frozen precipitation was then computed, again with the logit model, as a function of these new variables. In order to get stable results in this last step, data for all stations were combined. In addition to the meteorological variables, we also used the first harmonic of the day of year and station elevation as predictors.

2. DEFINITION OF PREDICTAND

For our purposes, "frozen" precipitation is defined as some form of snow or sleet (ice pellets); freezing rain and mixed rain and snow are included with rain and drizzle in the "unfrozen" category. The observations we had available did not allow a more definitive breakdown or the definition of other categories such as "mixed rain and snow" or "freezing precipitation." For simplicity, in this paper the terms snow (rain) and frozen (unfrozen) precipitation will be used interchangeably.

3. DERIVATION OF 50 PERCENT VALUES

The use of an equal-probability thickness value for differentiating between liquid and frozen precipitation is not new. Wagner (1957) determined equal-probability values for 1000-500 mb thickness several years ago, and his results have been used extensively. He developed a value for each of 40 stations in the conterminous U.S. from two seasons of data. For our study, we had available three seasons of data: September 1969 through April 1970, September 1970 through April 1971, and September 1971 through March 1972. Since we were using forecasts at grid points instead of calculated thicknesses at upper air observing stations, we could interpolate and use values at all locations where we had surface reports of precipitation type. Although we had surface reports at 234 stations, only 186 of the station records contained sufficient cases of frozen precipitation for reliable 50-percent values to be determined. These 186 stations are shown in Fig. 1 along with the other 48 stations for which very few, if any, observations of snow were available.

We used the logit model (Brelsford and Jones, 1967 and Jones, 1968) to determine the

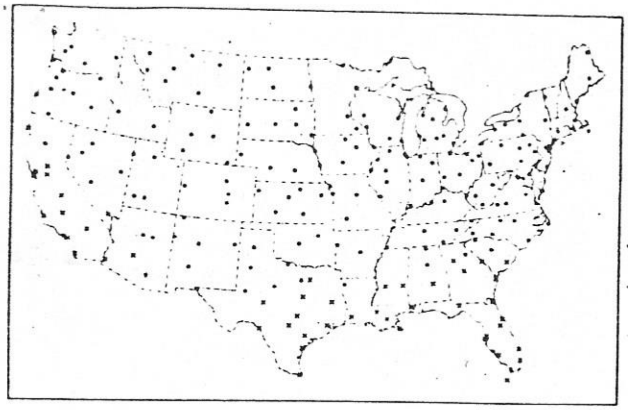


Figure 1. The 234 stations for which observations were available. The 48 stations for which 50-percent values could not be determined statistically are shown by X's.

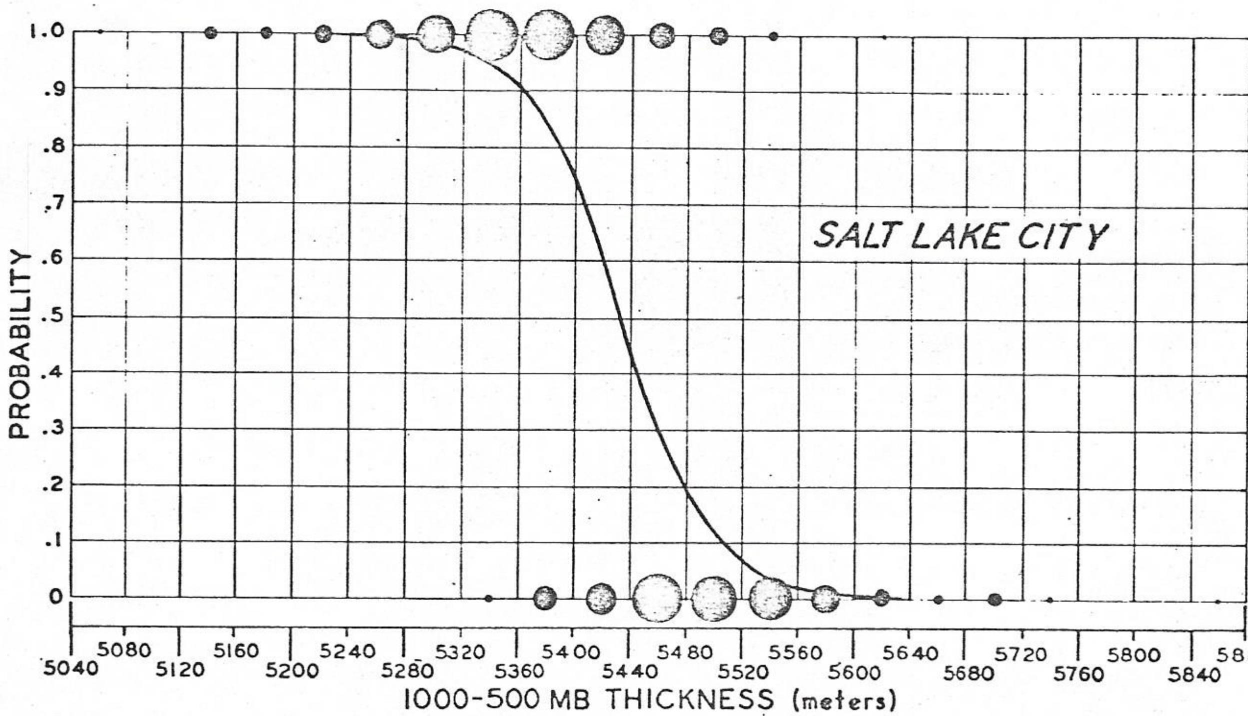
50-percent values of three variables, 1000-500 mb thickness, 850-mb temperature, and B.L. temperature, all of which were forecast by the NMC PE numerical model. The logit model provides a means of fitting a sigmoid or S-shaped curve when the dependent variable (Y) is binary and the independent variable (X) is quasi-continuous. From it, the probability of the binary variable having the value of one, say, can be expressed

$$P[Y=1|X] = \frac{1}{1 + \exp(a + bX)} .$$

Our computer program determines the maximum likelihood estimates for the model parameters a and b.

We will use 1000-500 mb thickness as an example of how the 50-percent values of the three variables were found. Surface observations were available at 6-hourly intervals, as were the forecasts from the PE model. The thickness forecasts valid at the same time the observations were made were interpolated to the station locations. Forecasts for four projection times--6, 12, 18, and 24 hours--from both the 0000 and 1200 GMT PE model runs were combined into one sample. This pooling of data was done to insure that the sample contained an adequate number of snow cases for reliable estimation. The resulting sample sizes ranged from 90 at Tonopah, Nevada to 1600 at Traverse City, Michigan.

After the logit model constants were derived, it was a simple matter to find the value of thickness for which the probability of snow is 50-percent; this value is given by -a/b. A typical example of the analysis is shown in Fig. 2. The resulting 50-percent value of 1000-500 mb



SNOW CASES	*1*	*0*	*7*	*6*	*9*	*20*	*37*	*74*	*73*	*44*	*16*	*8*	*4*	*0*	*1*	*0*	*0*	*0*	*0*	*0*	*0*
TOTAL CASES	*1*	*0*	*7*	*6*	*9*	*20*	*37*	*77*	*88*	*69*	*84*	*68*	*53*	*18*	*9*	*4*	*7*	*2*	*0*	*0*	*1*
RELATIVE FREQUENCY	*1.00*	—	*1.00*	*1.00*	*1.00*	*1.00*	*1.00*	*.96*	*.83*	*.64*	*.19*	*.12*	*.08*	*.00*	*.11*	*.00*	*.00*	*.00*	—	—	*.00*
LOGIT PROBABILITY	*1.00*	*1.00*	*1.00*	*1.00*	*1.00*	*1.00*	*1.00*	*.95*	*.84*	*.59*	*.29*	*.11*	*.03*	*.01*	*.00*	*.00*	*.00*	*.00*	*.00*	*.00*	*.00*

Figure 2. Probability of frozen precipitation as a function of 1000-500 mb thickness at Salt Lake City, Utah. The number of snow cases, precipitation cases, relative frequency of snow, and logit probability in 40-meter intervals of thickness are shown. The areas of the dots on the graph represent the number of cases in the 40-meter intervals.

thickness for Salt Lake City is 5432 m.

The 50-percent values for all stations were subjectively analyzed in two ways. First, all values were plotted on a map and viewed for consistency with neighboring values. Secondly, all values were plotted on a graph, shown in Fig. 3, as a function of station elevation. It is apparent from Fig. 3 that the points fall into 3 groups: (1) those with a strong marine influence, (2) those not included in (1) west of the continental divide, and (3) those not included in (1) east of the continental divide.

The stations labeled as having a strong marine influence are all in the Pacific Northwest, west of the Cascades, except two--Cape Hatteras, North Carolina and Nantucket, Massachusetts. These stations are situated such that low-level air trajectories will usually have a recent overwater history. In those precipitation cases when the 1000-500 mb thickness indicates a 50-50 chance of snow, the temperature sounding will exhibit a steeper lapse rate at stations with a strong marine influence than at other stations. Therefore, for a given low-level temperature, which is the controling factor in whether precipitation will be rain or snow, the 50-percent thickness value at Seattle is lower than at, say, Washington, D. C.

Most stations west of the continental divide, especially those in the northern latitudes, exhibit lower 50-percent values than stations at similar altitudes east of the continental divide. This probably indicates western stations have a steeper lapse of temperature than eastern stations in cases when rain and snow are about equally likely. Another possible explanation is that the PE model produces forecasts with different biases in the West than in the East.

The logit model did not give acceptable 50-percent values for two stations, Portland and Medford, Oregon, even though each station had quite a number of both rain and snow cases. For these two stations, a subjective estimate was made from the original station data. These subjective estimates were noted to be consistent with values at surrounding stations.

The problem at these two stations was caused by the occurrence of an unusual number of snow cases at rather high thickness values. The model, in attempting to fit all the data (rather than data only in the vicinity of the 50-percent value) gave a poor fit near the 50-percent value. Evidently, the symetric nature of the curve is a severe restriction in some cases.

The 50-percent values for 850-mb temperature and B.L. temperature were found in a similar manner. The graph for 850-mb temperature, similar to Fig. 3, showed the same three groupings of stations as did the graph for 1000-500 mb thickness, although the groups were not as distinct. The graph for B.L. temperature showed very little recognizable grouping of stations. There was some indication that stations in northeastern Texas, Arkansas, and western Tennessee had lower values (about -1.0C) than other stations of similar elevation (about +5.0C). This may indicate that snow occurs in this area with different synoptic conditions than for other areas. On the other hand, snow occurs there only infrequently, and these values are somewhat in question.

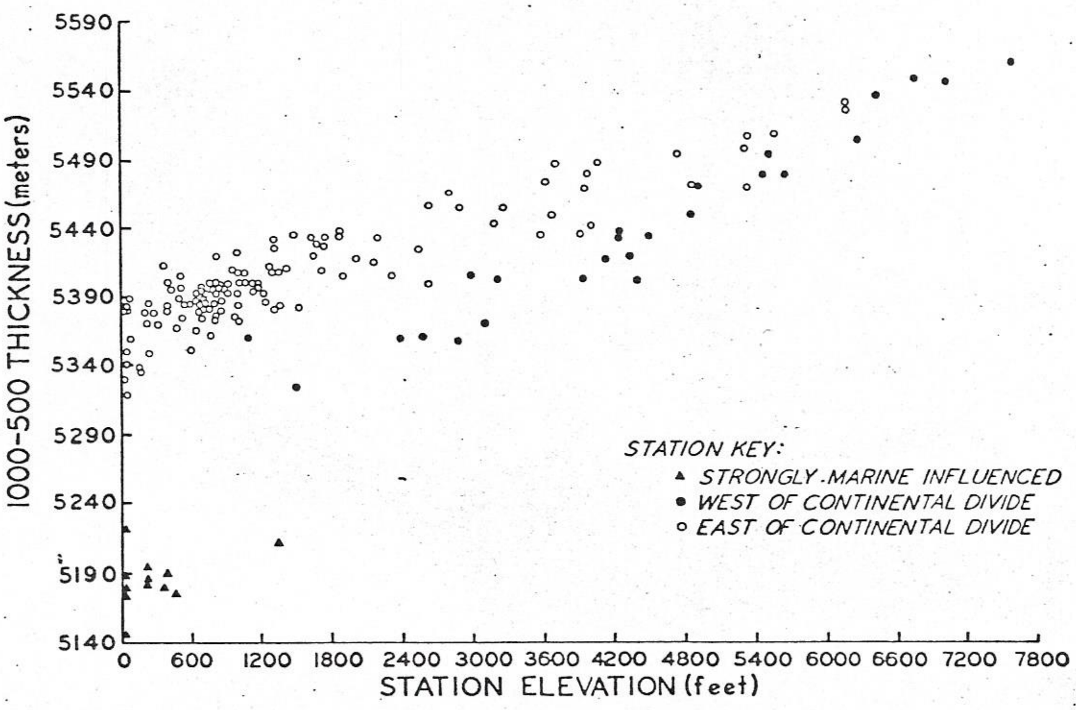


Figure 3. 50-percent values of 1000-500 mb thickness as a function of station elevation.

4. DERIVATION OF PREDICTION EQUATIONS

The logit curves described in the preceeding section could be used to estimate the conditional probability of snow in an operational environment. However, each curve uses only one independent variable, and some method would be needed to combine estimates from the three curves at each station. Also, for many stations, the estimates would not be very reliable for low and high probabilities, even though the distribution of snow-rain cases was such as to allow the determination of a 50-percent value. Further, no estimates could be made for the stations with insufficient snow cases to determine a reliable logit curve or 50-percent value.

The logit model provides a means for obtaining estimates based on more than one independent variable. The estimation equation can be written

$$P[Y=1|X_1,X_2,\ldots,X_n] = \frac{1}{1+\exp(a+b_1X_1+b_2X_2+\ldots+b_nX_n)}$$

for n independent variables.

In order to provide an adequate data sample for fitting the constants (a and b's) in the above equation, the assumption was made that a particular deviation from a station's 50-percent value will produce the same probability of snow (or rain) at all stations. For instance, Fig. 2 shows that a 1000-500 mb thickness of 5362 m at Salt Lake City indicates about a 90 percent (conditional) probability of snow. This is 70 m below the critical thickness of 5432 m. Our assumption is that a thickness 70 m below the critical value at any station will indicate about a 90 percent probability of snow. (The 70 m value at Salt Lake City is used for illustration only; a better value can be determined from a sample composed of data from many stations). This assumption allows data from all stations for which we have a 50-percent value to be combined into one sample. At each station, the 50-percent value of each of 1000-500 mb thickness, 850-mb temperature, and B.L. temperature was subtracted from the original record of the corresponding variable. This resulted in a large sample in which the independent variables were expressed as deviations from a station "constant."

In addition to the three meteorological variables, we also used the station elevation and the sine (SIN) and cosine (COS) of the day of the year as independent variables. These latter are expressed as:

$$\text{SIN} = \text{Sin}\ \frac{2\pi i}{365},$$

$$\text{COS} = \text{Cos}\ \frac{2\pi i}{365} \quad \text{where } i = 1 \text{ for Jan. 1, etc.}$$

It is not necessary to use predictions of the meteorological variables valid at the same time as the PoFP(P) forecast. For instance, if we want a probability forecast 36 hours after the 0000 GMT PE run time, we can use the PE B.L. temperature forecast valid 48 hours after run time if we so desire, or we can even use the B.L. temperature forecast valid at 36 hours and 48 hours after run time if we wish.

Separate logit forecast equations were derived for each of the PE run times, 0000 and 1200 GMT, and for each of 4 projections, 12, 24, 36, and 48 hours. Table 1 gives the predictors used for each projection. Projection, in each case, refers to the number of hours after PE run time.

The logit computer program does not provide for objective selection of a few predictors from a large group. However, a number of runs were made which constituted, in effect, a subjective screening. Addition of more predictors to the final set selected is unlikely to give substantially better results. In general, the PE predictors valid at the same time as the PoFP(P) forecast were the most important. The addition of PE predictors valid 12 hours before and after the time of the PoFP(P) forecast was also important. The 850-mb temperature was the best of any single predictor by a considerable margin. An additional predictor, the 1000-850 mb

TABLE 1. Predictors in the PoFP(P) equations for each of the 4 projections.

12-Hr Forecast		24-Hr Forecast		36-Hr Forecast		48-Hr Forecast	
Predictor	Projection	Predictor	Projection	Predictor	Projection	Predictor	Projection
Station Elevation	-	Station Elevation	-	Station Elevation	-	Station Elevation	-
Sin DOY	-	Sin DOY	-	Sin DOY	-	Sin DOY	-
Cos DOY	-	Cos DOY	-	Cos DOY	-	Cos DOY	-
1000-500 mb Thick.	12	850 mb Temp.	12	850 mb Temp.	24	850 mb Temp.	36
850 mb Temp.	12	B.L. Temp.	12	B.L. Temp.	24	B.L. Temp.	36
B.L. Temp.	12	1000-5-- mb Thick.	24	1000-500 mb Thick.	36	1000-500 mb Thick.	48
850 mb Temp.	24	850 mb Temp.	24	850 mb Temp.	36	850 mb Temp.	48
B.L. Temp.	24	B.L. Temp.	24	B.L. Temp.	36	B.L. Temp.	48
		850 mb Temp.	36	850 mb Temp	48		
		B.L. Temp.	36	B.L. Temp.	48		

thickness, was tried and discarded since it was of no use in improving the relationships. The station elevation and day of year variables were of marginal benefit.

The P-scores (Brier, 1950) on the dependent data for the 8 prediction equations are shown in Table 2. As expected, scores increased (forecasts became poorer) with projection. There is an indication that forecasts made from 0000 GMT data are better than those made from 1200 GMT data for the same projection.

Table 2. P-scores for PoFP(P) forecasts on the 3 years of dependent data.

Projection (Hr)	Initial Data Time 0000 GMT	1200 GMT
12	.107	.118
24	.124	.125
36	.140	.159
48	.177	.179

The constants and coefficients derived for the logit prediction equations are difficult to interpret for at least two reasons. First, because of the complex interrelationships among the several variables, the effect of individual terms cannot be adequately assessed. Secondly, the logit is non-linear in form, and this complicates the interpretation of individual predictors. Therefore, the actual equations are not shown here.

5. OPERATIONAL ASPECTS

The objective forecast method described above was completed in March 1973. However, on November 1, 1972, a similar system was made operational by the NWS. It differed from the one described above in only one important respect--it was derived on two seasons of data rather than three, the two seasons being 1969-70 and 1970-71. A minor difference is that the 50-percent values were determined in a slightly different way and were determined for 182 rather than for 186 stations. Before implementation in November, we tested the 12-hour equation derived from the 0000 GMT PE run on independent data--the 1971-72 winter season. The P-score on the independent data was .109; this compares favorably with the dependent data P-score of .110.

The operational products include a 4-panel facsimile chart graphically portraying probability of precipitation (PoP) and PoFP(P) forecasts. The PoFP(P) forecasts are valid 12, 24, 36, and 48 hours after PE run time; the PoP forecasts are valid for four 12-hour periods beginning 12, 24, 36, and 48 hours after PE run time. An example of one panel from the facsimile chart is shown in Fig. 4.

A teletype bulletin, a portion of which is shown in Fig. 5, is disseminated over "Service C." The bulletin gives specific values of PoFP(P) and PoP at 154 cities for the same forecast projections as given in the facsimile chart. Forecasts for other stations are available through the Weather Message Switching Center at Kansas City, Missouri on a request/reply basis.

In order that forecasts could be made for all stations and the facsimile maps of the U.S. would not have blank areas, 50-percent values were estimated at those stations for which such values could not be determined statistically. In this estimation, we took into account: (1) the relationship of 50-percent value to station elevation, such as shown in Fig. 3; (2) values at neighboring stations; (3) nearness to a large body of water; and (4) most likely direction of low-level winds in snow-threat situations. Forecasts for the stations where it was necessary to make these estimates will undoubtly be less reliable than forecasts for stations with statistically-derived 50-percent values. However, it is felt that even these less-reliable forecasts will be of use in alerting forecasters to snow-threat situations.

We have not yet completed a verification of the operational product for the 1972-73 winter season. However, there is no reason to believe it will not hold up well. One sequence of sample forecasts is shown in Fig. 6.

Fig. 6. depicts PoFP(P) forecasts made from the 0000 GMT PE run of February 8, 1973. The occurrence of snow and rain at the valid time is indicated by stippling and vertical line shading respectively. The 12-hour PoFP(P) forecasts shown in Fig. 6(a) are valid at the same time as the PoFP(P) forecasts shown in Fig. 4. It should be noted that 5-, 50-, and 95-percent PoFP(P) isopleths are shown in Fig. 6(a) while 10-, 50-, and 90-percent isopleths are shown in Fig. 4. The observed areas of precipitation in Fig. 6(a) and 6(b) correspond respectively to the beginning and end of the 12-hour period for which PoP forecasts are shown in Fig. 4.

In the case depicted in Fig. 6, considerable change in temperature occurred over the eastern U.S., and an unusually large amount of snow was received in Georgia and North Carolina. This figure shows that (1) no rain occurred in areas where the conditional probability of rain was 5 percent or less, (2) no snow occurred in areas where the conditional probability of snow was 5 percent or less, and (3) the actual rain-snow line was usually rather close to the 50-percent probability line.

6. SUMMARY AND CONCLUSIONS

The system we have developed for forecasting the conditional probability of frozen precipitation shows much promise of furnishing substantial guidance to local forecasters. It is an extention of the successful work of Wagner (1957) in four ways: (1) several predictors were used rather than only 1000-500 mb thickness; (2) 186 stations were used instead of 40; (3) forecasts from the PE model valid at or near the time of the desired forecast were used instead of observations, and (4) three years of data were used rather than two.

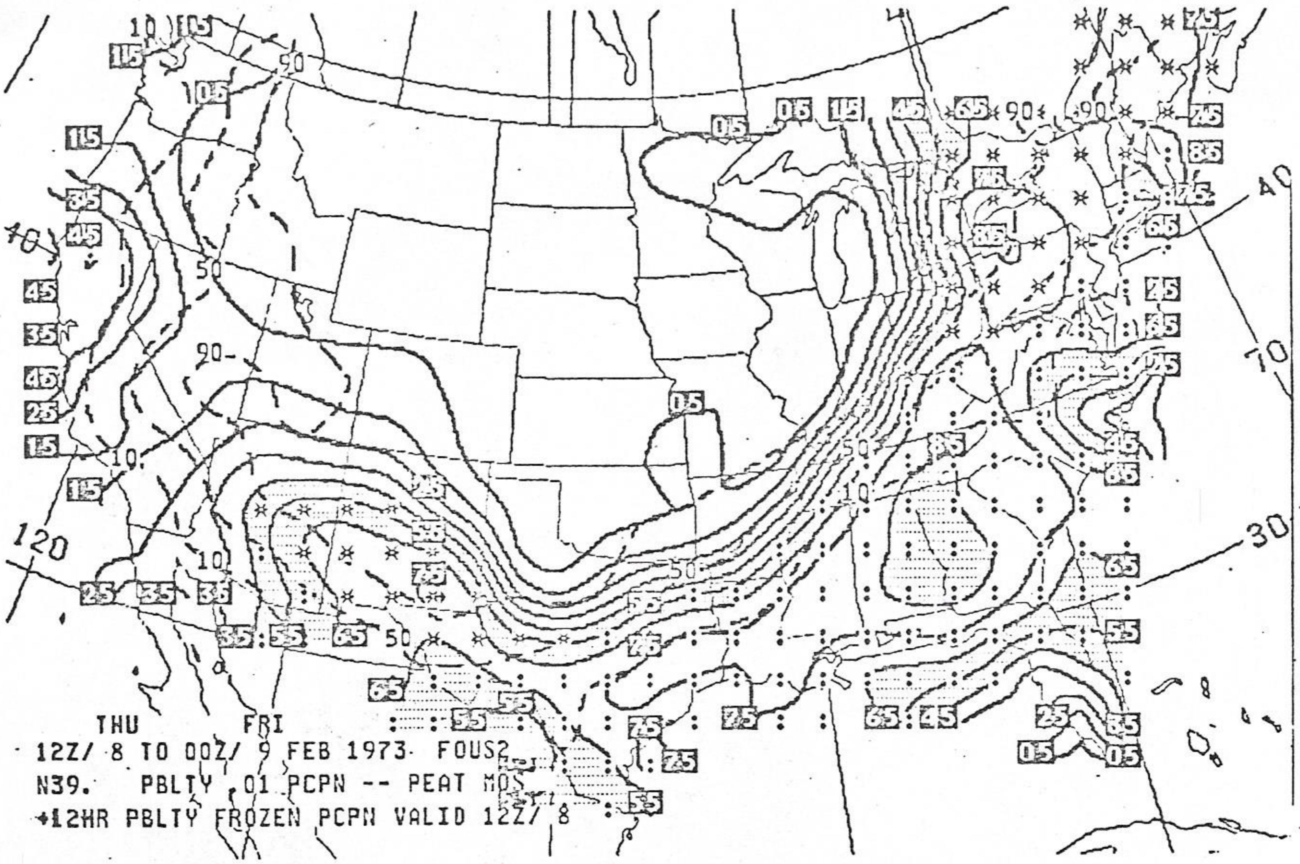


Figure 4. An example of a panel from an operational facsimile chart showing PoP and PoFP(P) forecasts made from the 0000 GMT February 8, 1973 PE run. The PoP forecasts, shown as solid lines at 10-percent intervals, are valid for the period 1200 GMT February 8 to 0000 GMT February 9. The PoFP(P) forecasts shown as dashed lines for values of 10, 50, and 90 percent, are valid at 1200 GMT February 8. The areas defined by PoP isopleths 45 to 65-percent and above 85-percent are shaded. Areas where snow or rain may be expected are indicated by asterisks or dots respectively.

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PEATMOS POP AND POFP/P

	POP				POFP/P					POP				POFP/P			
	1	2	3	4	12	24	36	48		1	2	3	4	12	24	36	48
CAR	80	70	30	20	92	95	99	99	BGR	90	80	30	10	60	75	98	99
BTV	70	40	5	20	83	99	99	99	MSS	70	20	5	20	96	99	99	99
PWM	60	80	20	10	30	73	98	99	CON	60	70	5	10	70	94	99	99
BOS	70	80	10	10	39	73	98	99	PVD	70	70	5	10	25	62	97	99
BDL	70	70	5	20	45	89	99	99	NYC	60	70	5	10	20	71	98	93
ALB	70	40	5	20	73	98	99	99	SYR	80	30	20	20	85	99	99	99
BGM	80	40	20	20	83	99	99	99	IPT	80	40	20	10	63	98	99	99
HAR	80	60	30	10	35	91	99	99	PHL	70	80	5	5	16	65	98	99
ACY	70	70	5	10	3	37	93	99	DCA	70	80	5	10	11	55	97	99
EKN	80	40	20	10	45	95	99	99	PIT	80	30	10	10	77	99	99	99
BFD	80	30	20	10	97	99	99	99	BUF	80	20	20	10	98	99	99	99
SSM	20	20	40	40	99	99	99	99	HTL	0	5	30	20	99	99	99	99
GRR	0	5	20	10	99	99	99	99	DTW	10	5	5	20	99	99	99	99
CLE	40	5	5	10	95	99	99	99	TOL	20	5	5	20	93	99	99	99
SBN	0	5	10	10	99	99	99	98	IND	10	5	10	10	99	99	99	99
DAY	30	5	2	10	95	99	99	99	CMH	60	10	0	10	91	99	99	99
SDF	50	5	0	10	83	99	99	99	HTS	80	30	20	10	32	97	99	99
CRW	80	40	20	10	22	93	99	99	ROA	80	50	5	2	13	67	98	99
RIC	80	80	5	10	6	24	90	98	ORF	50	80	10	20	2	5	50	87
GSO	80	80	5	2	4	21	92	98	AVL	80	60	5	10	18	81	98	99

Figure 5. A portion of the operational teletype bulletin transmitted over Service C showing specific values of PoP and PoFP(P) for the PE run time of 0000 GMT February 8, 1973. The PoP forecasts are valid for four 12-hour <u>periods</u> beginning at 12, 24, 36, and 48 hours after PE run time. The PoFP(P) forecasts are valid at the <u>specific times</u> 12, 24, 36, and 48 hours after PE run time.

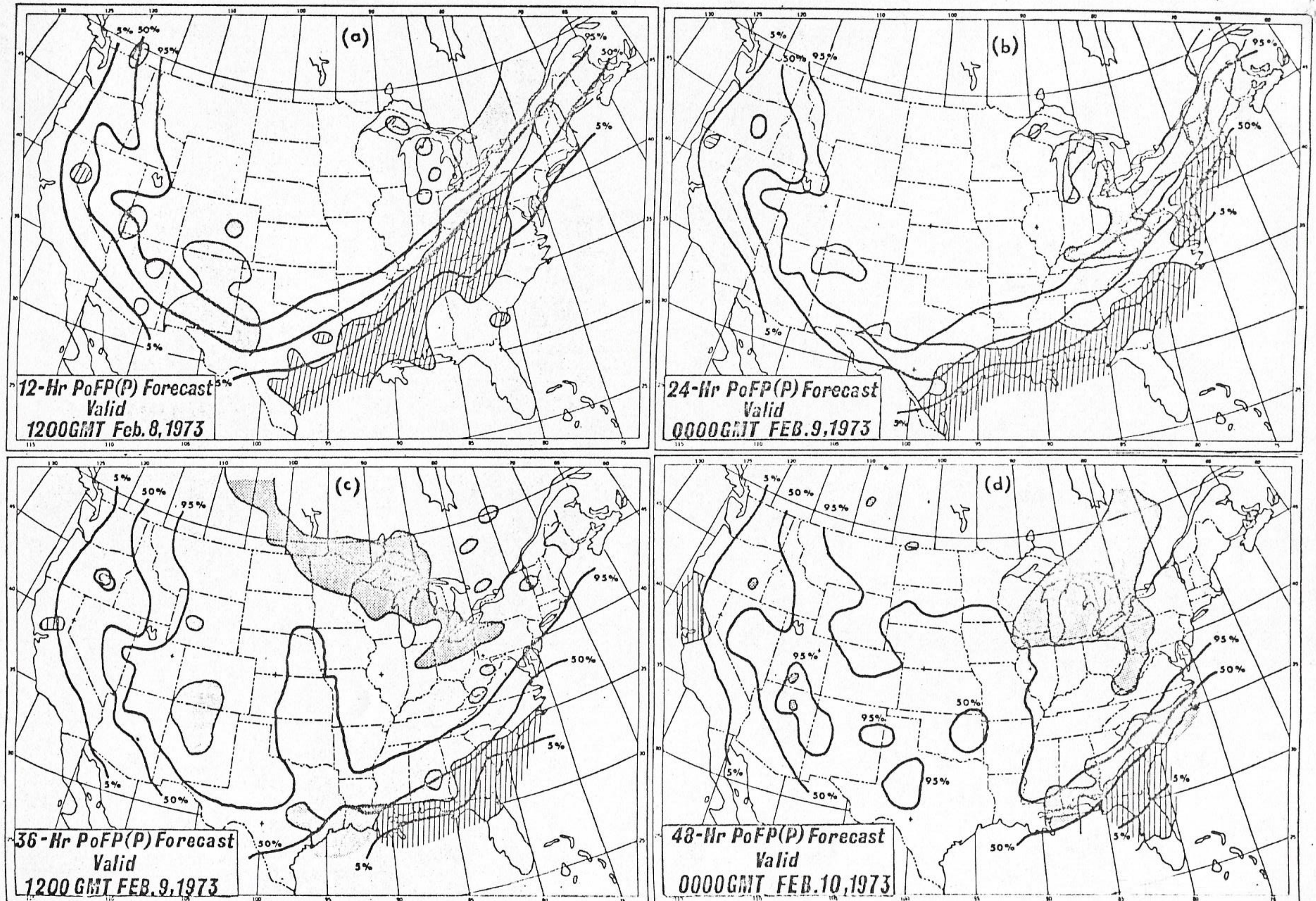

Figure 6. PoFP(P) forecasts made from the 0000 GMT PE run of February 8, 1973, valid at (a) 12-, (b) 24-, (c) 36-, and (d) 48-hour projections. Probability lines of 5-, 50-, and 95-percent are shown. The occurrence of snow and rain at the valid time is indicated by stippling and vertical line shading respectively.

We believe this is the first significant use of the logit model in meteorology; with very few exceptions, it performed beautifully.

Accuracy of objective PoFP(P) forecasts can be improved in several ways, including the following, listed in order of importance: (1) improved forecasts of temperature and thickness from numerical models; (2) development of unique prediction equations for a number of "homogenous" areas of the U.S., rather than development of only one equation for the whole area; (3) determination of better 50-percent values through use of a larger data sample.

The usefulness of the guidance forecasts will be enhanced when we can deal with additional categories, such as "freezing precipitation" and "sleet." A possible category of "mixed rain and snow" is of doubtful importance, since a probability of snow near 50-percent is in itself a tipoff to possible rain and snow mixed. Categories of precipitation type other than those used in this study may have to be dealt with differently, since a simple logit curve may not be adequate to fit the data.

7. ACKNOWLEDGMENTS

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