II.2-SNOW-43 NWS-43 SNOW MODEL

Introduction

The SNOW-43 Operation is a state-space version of the snow accumulation and ablation model first described in NOAA Technical Memo HYDRO-17 (Anderson, 1973) with the option of performing Kalman filtering updating.

The state-space formulation and Kalman filter implementation are described in NOAA Technical Report NWS 43 (Day, 1990). The Kalman filter accounts for the relative uncertainties of observed and simulated water-equivalent values in a procedure that optimally updates the model simulated states using areal estimates of snow-water-equivalent based on observations.

The snow model uses inputs of precipitation and temperature to simulate the snow accumulation and ablation processes. The model keeps continuous account of state variables needed to describe the snow cover, e.g., frozen and liquid snow-water-equivalent, snow cover heat storage, an index temperature of the pack and the areal extent of snow cover. Uncertainties in the estimates of these model states build up during the course of a snow season as a result of both model and data errors. Model error is introduced because of an inability to perfectly represent the physical processes integrated over a basin. Data errors are introduced due to measurement errors and the inability to adequately estimate meteorological inputs on an areal basis.

Observations of snow-water-equivalent provide an additional source of information about the water balance of a basin. The SNOW-17 model includes the ability to update the water-equivalent model states with observations of areal snow-water-equivalent by replacing the model water-equivalent states with external estimates based on observations. These external estimates also contain uncertainty, however and may or may not be closer to the true physical conditions than the simulated states. The information available from both the simulation and the observations can be used more effectively by optimally combining these estimates based on their respective uncertainties. The ability to appropriately update the snow model states can have a significant positive impact on the simulation accuracy of runoff peaks and volumes from a basin.

The Kalman filtering updating procedure which was described in NWS 43 optimally combines estimates of snow water-equivalent from observations with the simulated states generated by the SNOW-17 model. In addition, the National Operational Hydrologic Remote Sensing Center (NOHRSC) has implemented the NWS Snow Estimation and Updating System (SEUS) (McManamon et al., 1993) to provide operational estimates of snow-water-equivalent. The SNOW-43 Operation is an implementation of the Kalman filtering procedure that takes advantage of the operational estimates of snow water-equivalent generated by SEUS. This Operation was designed to reproduce the functionality of the SNOW-17 snow accumulation and ablation Operation with the addition of Kalman filtering updating capabilities.

Kalman Filtering Updating

An optimal estimator processes measurements to produce a minimum error estimate of the state of a system by utilizing knowledge of system errors, measurement errors and initial condition information (Gelb et al., 1974). The Kalman filter (Kalman, 1960 and Kalman and Bucy, 1961) is an optimal estimator which provides a linear unbiased minimum variance estimate of a model state. In order to apply the Kalman filter, the system and measurement dynamics must be expressed as linear functions of the model states. In discrete form, the system equation expresses the state vector at time t+1 as a function of the state vector at time t and external driving forces. The measurement equation relates the state vector to the observations. Numerous sources describe the formulation and derivation of the filter equations (Gelb et al., 1974 and Jazwinski, 1970).

The discrete form of the Kalman filter is:

System equation:

$$x_t=\phi_{t-1}x_{t-1}+G_{t-1}u_{t-1}+\Gamma_{t-1}w_{t-1}$$

Measurement equation:

$$z_t = H_t x_t + v_t$$

where x_t is the state vector at time t, (n x 1)
- z_t is the measurement vector at time t, (m x 1)
- u_t is the input vector at time t, (® x 1)
- w_t is the system noise vector at time t, (p x 1)
- v_t is the measurement noise vector at time t, (m x 1)
- ϕ_t is the system transition matrix at time t, (n x n)
- G_t is the input weighting matrix at time t, (n x r)
- Γ_t is the system noise weighting matrix at time t, (n x p)
- H_t is the state/observation weighting matrix at time t, (m x n)
- m is the number of measurements
- n is the number of states
- r is the number of inputs
- p is the number of random inputs

System and measurement noise are assumed to be independently and identically distributed Gaussian random variables with the following properties:

$$E(w_t)=0$$
$$E(v_t)=0$$
$$E(w_t w_k^T)=Q\delta_{tk}$$
$$E(v_t v_k^T)=R\delta_{tk}$$
$$E(w_t v_k^T)=0$$
for all t, k

where Q is the system error covariance matrix,
R is the measurement error covariance matrix and
δ_{tk} is the Kronecker delta function such that:

$\delta_{tk} = 1$ for $t = k$

$\delta_{tk} = 0$ for $t \neq k$

The solution to the filter can be derived by assuming that the optimal estimator is a linear combination of the optimal state estimate before an observation and of the observation itself. The requirements that the estimator be unbiased and possess minimum variance lead directly to the solution:

$$\hat{x}_{(t+1)/t}=\Phi_t\hat{x}_{t/t}+G_t u_t$$

$$P_{(t+1)/t}=\Phi_t P_{t/t}\Phi_t^T+\Gamma_t Q_t \Gamma_t^T+G_t U_t G_t^T$$

$$K_{t+1}=P_{(t+1)/t}H_{t+1}^T(R_{t+1}+H_{t+1}P_{t+1/t}H_{t+1}^T)^{-1}$$

$$\hat{x}_{(t+1)/(t+1)}=\hat{x}_{(t+1)/t}+K_{t+1}(z_{t+1}-H_{t+1}\hat{x}_{(t+1)/t})$$

$$P_{(t+1)/(t+1)}=(I-K_{t+1}H_{t+1})P_{(t+1)/t}$$

where $\hat{x}_{\frac{t}{s}}$ is the estimate of state vector at time t, given information at time s, (n x 1)

$P_{\frac{t}{s}}$ is the state estimate error covariance matrix at time t, given information at time s, (n x n)

U_t is the input noise covariance matrix, (® x r)

K_t is the Kalman gain matrix at time t, (n x m)

is the identity matrix, (n x n)

Application to the Snow Model

Application of the Kalman filter requires the development of a linear system equation that describes the changes to model states in time and as a function of inputs, as well as a linear measurement equation that relates model states to observations. The first step in the development of these equations is to express the model in state-space form. State-space form requires that the model be reformulated from conditional execution logic typically used in computer programs to a single set of equations that express the changes in model states as a function of the current states and inputs.

The NWS snow accumulation and ablation model is extremely nonlinear and like many conceptual models, it makes frequent use of thresholds to indicate when the operating rules of the model change. The extended Kalman filter allows a nonlinear system to be linearized about some nominal state vector, which is usually selected to be the current estimate of the conditional mean of the state vector. Five model states were selected for inclusion in the state-space formulation. They are frozen water-equivalent (WE), negative heat storage (NEGHS), liquid water-equivalent (LIQW), snow cover temperature index (TINDEX) and areal extent of snow cover (AESC). Temperature and precipitation are the model inputs represented by the input vector, u.

Zero-one integer variables were used to indicate when the model switches from one mode to another as a result of crossing one of the model thresholds. The state-space equations were formulated in terms of these variables and the relationships representing these modes of operation. Partial derivatives were computed for the state-space equations and used in first-order Taylor series approximations to produce a linear set of equations for the changes in model states at a given time step. In practice, the complete nonlinear state-space equations are used to forecast the states and the stepwise linearized equations are only used to propagate the state error covariance matrix.

The measurement equation must also be written as a linear function of the states. The measurements in this case are estimates of snow-water-equivalent based on snow course, SNOTEL and flight-line observations. These observations are processed external to the filter to form a single measurement of the model snow-water-equivalent states. H is simply a vector that equates the measurement to the sum of the frozen and liquid-water-equivalent states.

For a given time step, the complete nonlinear state-space equations are used to forecast the states at time t + 1 given the state vector at time t and inputs for time period t. The filter is used to propagate the state error covariance matrix using the state-space equations which have been linearized around the conditional mean of the state vector. If an observation of water-equivalent is available at time t + 1, the Kalman gain is calculated and the states for time period t + 1 are updated. The state error covariance matrix is then updated, the time is incremented and the procedure is repeated for the next time step.

Features

One of the objectives in the development of the SNOW-43 Operation was to maintain the complete functionality of the SNOW-17 Operation. This objective was especially important in the development of the input structure, the functioning of run-time MODs and the incorporation of several features that were not included in the original development work. The structure of the input was patterned after the SNOW-17 input structure and requires that 'PROP' be specified as an option on the first card to activate the reading of additional cards containing the Kalman filtering parameters. PROP stands for 'propagate state error covariance matrix'. This structure allows existing SNOW-17 input to be used in the SNOW-43 Operation without any modification. MODs which are available in the SNOW-17 Operation include MFC, RAINSNOW, WEADD, WECHNG and AESCCHNG. These MODs are available in the SNOW-43 Operation. The WEADD, WECHNG and AESCCHNG MODS required special handling in the Operation since they directly modify model states whose variance is propagated with the filter. When these MODs are used in SNOW-43, the effected model states are updated outside of the filter and the state error covariance matrix is simply adjusted based on the magnitude of the changes to the model states.

If a RSNWELEV Operation is used to compute a rain-snow elevation time series, the SNOW-43 Operation accounts for the uncertainty in estimating the fraction of snow over the basin. The Operation estimates the uncertainty in the fraction of precipitation which is snow using the lapse rate defined in the RSNWELEV Operation, the error variance of the temperature input and the slope of the area-elevation curve.

The state-space and filtering equations were originally developed to be applicable for a six-hour computational time interval. In order to provide the same functionality as the SNOW-17 Operation, the software was modified to allow precipitation time intervals other than six hours and to allow temperature time intervals greater than the precipitation time interval. In all cases the state error covariance matrix is propagated at the time interval of the precipitation data.

Updating Options

The updating procedures can be turned on or off at program execution by means of the UPDATE flag in the calibration program or by using the IUPSC and IUPWE Techniques in the Operational Forecast Program. If the updating procedures are on, updating will occur whenever observed data are available. The SNOW-43 Operation supports three types of updating with snow-water-equivalent observations: replacement, gain and filter updating. In replacement updating, the model states are replaced with the observation. The SNOW-17 Operation only supports replacement updating. In gain updating, the updated snow-water-equivalent is computed by weighting the observation with a user-specified gain which can range from zero to one and weighting the simulated state with a weight of one minus the gain. In filter updating, the Kalman filter computes a gain based on the relative uncertainties of the observed and simulated values of water-equivalent. Since filter updating requires an estimate of the state uncertainties, filter updating can only occur when the PROP option is selected. Like SNOW-17, SNOW-43 currently can only update with areal extent of snow cover observations using the replacement technique.

The SNOW-43 Operation can use the run-time MOD WEUPDATE. This MOD takes precedence over updating based on an observed water-equivalent time series, a WEADD MOD, or a WECHANGE MOD. Required input for the MOD is an observed water-equivalent value. Optional input is either a variance of the observed value or a gain to be applied to the observation. If neither a variance nor a gain is specified, a default variance must be available as a parameter of the Operation for updating to occur.

When replacement or gain updating are used, the areal extent of the snow cover is automatically adjusted based on the updated water-equivalent states and the areal depletion curve. When the water-equivalent states are updated within the filter, slight adjustments may occur to other model states, including the areal extent of cover. Options are available to retain the filter updated areal extent of snow cover state or to adjust the areal extent of snow cover state based on the updated water-equivalent states and the areal depletion curve in the same manner as occurs in SNOW-17. If the filtered state is retained, the areal depletion curve is adjusted to be consistent with the updated snow-water-equivalent and areal extent of snow cover states. The default for this option is to adjust the filtered state in the same manner as occurs in SNOW-17. This snow cover adjustment also occurs if water-equivalent is updated using the replacement or gain techniques.

If the SNOW-43 Operation is used to perform filter updating, observations must be entered through an observed water-equivalent time series or through the WEUPDATE MOD. Since filter updating requires the variance of the water-equivalent observation, a new time series data type (OWEV) was needed for the observed water-equivalent variance when observations are entered through a time series. Observations can only be entered into the Calibration System programs through time series. Since there is no preprocessor to write observed water-equivalent time series to the Operational Forecast System Processed Database, observations of snow water-equivalent will normally be entered in the Operational Forecast Program using MODs. An additional time series data type was added for the variance of simulated water-equivalent time series (SWEV). Simulated water-equivalent and variance of simulated water-equivalent time series can be generated from either the Calibration System or Operational Forecast System programs.

Input Requirements

Several parameters in addition to those used in SNOW-17 are required for Kalman filtering updating. These parameters include the input error covariance matrix, the system error covariance matrix and default monthly values for the variance of water-equivalent observations. The input error covariance matrix, denoted by U, is a 2 x 2 error covariance matrix of the model inputs, MAP and MAT (see Table 1). The covariance of MAP error with MAT error is assumed to be zero, so that the only required values are the diagonal elements, which are the MAP and MAT error variances. The MAP error variance is assumed to be proportional to the magnitude of MAP and the input is expressed as an error coefficient of variation. For each time step the MAP error variance is computed by multiplying the error coefficient of variation by the MAP value and squaring the result. The MAT error variance is directly specified.

Table 1. Input Error Covariance Matrix

	MAP	MAT
MAP	$U_{1,1}$	$U_{1,2}$
MAT	$U_{2,1}$	$U_{2,2}$

The system error covariance matrix, denoted Q, is a 5 x 5 matrix which represents the uncertainty that is added to the states when the model is used to predict the states at the end of a time step given the states at the beginning of the time step and the inputs. This error is the result of the model's inability to exactly account for all of the natural snow accumulation and ablation processes on an areal basis (see Table 2). The diagonal values of the matrix represent the error variances and the off-diagonal elements represent the error covariances. Normally, the off-diagonal elements are assumed to be insignificant and are assigned zeroes, but non-zero values may be specified. Since the matrix is symmetrical, only half of the off-diagonal elements need be entered. The magnitude of the values is a function of the size of the snowpack, the quality of the calibration and the ability of the model to accurately simulate snow accumulation and ablation in a particular basin. In general, one would expect larger system error values for basins with large errors between simulated and observed flows, however, errors in streamflow simulation include the effects of data errors (e.g., MAP and MAT) and other model errors (e.g., soil moisture accounting and unit hydrograph). The magnitude of these values is also a function of the model computational time step, since the system errors are added each time step.

Table 2. System Error Covariance Matrix

	WE	NEGHS	LIQW	TINDEX	AESC
WE	$Q_{1,1}$	$Q_{1,2}$	$Q_{1,3}$	$Q_{1,4}$	$Q_{1,5}$
NEGHS	$Q_{2,1}$	$Q_{2,2}$	$Q_{2,3}$	$Q_{2,4}$	$Q_{2,5}$
LIQW	$Q_{3,1}$	$Q_{1,2}$	$Q_{1,3}$	$Q_{1,4}$	$Q_{1,5}$
TINDEX	$Q_{4,1}$	$Q_{4,2}$	$Q_{4,3}$	$Q_{4,4}$	$Q_{4,5}$
AESC	$Q_{5,1}$	$Q_{5,2}$	$Q_{5,3}$	$Q_{5,4}$	$Q_{5,5}$

The measurement equation contains an error term which represents the error in using the observation to estimate the model states, denoted R. In this case, the observation is a preprocessed estimate of the model areal snow-water-equivalent states. Point and line observations of snow-water-equivalent are used to estimate actual basin areal snow-water-equivalent and then adjusted to account for any differences between actual basin water-equivalent and model areal water-equivalent. R represents the total uncertainty in using the preprocessed observation to estimate the model states. Default monthly values of R may be specified as SNOW-43 parameters. These defaults will only be used if observation error variances are not specified at program execution through a time series or a run-time mod.

Parameter Estimation

Effective use of the Kalman filtering updating technique in the SNOW-43 Operation requires accurate estimation of the associated model parameters. The updating procedure weights the observed and simulated values according to the relative uncertainties associated with the estimates. The error variance of observed water-equivalent values are generally defined outside of the model and input through a time series or MOD. The error variance of simulated water-equivalent states, however, are propagated by the model based on the system error covariance matrix and input error covariance matrix parameters. If these parameters do not accurately reflect the actual level of error, a less than optimal combination of updated water-equivalent will result. For example, if the model's estimate of the simulated water-equivalent error variance is too high, the observed water-equivalent value will be given too much weight in an update.

Although limited experience exists in the application of SNOW-43, a procedure is presented for parameter estimation that was applied to three basins: Black River at Point Pines, Blue River at Dillon Reservoir and North Fork of the Clearwater River at Dworshak Reservoir. Initial values for all the parameters were based on expected magnitudes of errors in the different components, i.e., input errors and system errors. The input errors are a function of the network density and its ability to represent a basin average. In a mountainous area 0.2 was selected as a reasonable estimate of the coefficient of variation for a 6-hour MAP value and 1.0 (DEGC squared) was selected as a reasonable estimate of the error variance for a 6-hour MAT value.

It was assumed that the off-diagonal elements of the system error covariance matrix were zero. This implies that the system errors introduced into the different states are not correlated. Although some correlation likely exists, these off-diagonal elements were considered less significant. Initial values of the diagonal elements were based on the expected magnitude of error that would be introduced by the model in each state over a 6-hour time period. During the original development work on the Animas Basin at Durango, Colorado, the state error covariance matrix, P, was monitored throughout the course of a snow accumulation and ablation season and the initial values of Q were adjusted so that they produced what were considered to be reasonable values in the P matrix. Table 3 shows the system error covariance matrix values which were derived for the upper subarea of the Animas Basin at Durango, Colorado. These values were used as initial values for all basin subareas.

Table 3. Example of Initial Estimate of System Error Covariance Matrix (Q)

	WE	NEGHS	LIQW	TINDEX	AESC
WE	8.5	0	0	0	0
NEGHS	0	0.01	0	0	0
LIQW	0	0	0.01	0	0
TINDEX	0	0	0	0.01	0
AESC	0	0	0	0	0

Since updating is being done with observations of water-equivalent, the most important state error variance is the one corresponding to the frozen water-equivalent state, $P_{1,1}$. The error variance of the frozen water-equivalent is most sensitive to the element of the system error covariance matrix ($Q_{1,1}$) corresponding to the frozen water-equivalent state. $Q_{1,1}$ was calibrated for each basin subarea based on an analysis of the simulated and pseudo-observed water-equivalent values for a particular date in the spring corresponding to maximum seasonal snow accumulation. The pseudo-observed values are estimated using the Snow Estimation and Updating System (SEUS). These values are the best available estimates of the true model snow-water-equivalent states. It was assumed that the mean squared error between the simulated and pseudo-observed water-equivalent values was a reasonable estimate of the error variance of the frozen water-equivalent state for this date. $Q_{1,1}$ was adjusted until $P_{1,1}$ matched the expected value for a typical year. The procedure could be repeated for additional years and values averaged over all years.

Some additional work was done to determine the updating procedure's sensitivity to the updating parameters. $Q_{1,1}$ was optimized first for the upper and then for the lower subarea using the root mean squared error in daily streamflow (DRMSE) as an objective function. The MAP and MAT error variances were optimized in the same manner, fixing the $Q_{1,1}$ elements of the system error covariance matrices at their optimum values. Care must be taken that this optimization step does not produce unrealistic values. When this procedure was applied to the three basins, the optimization step did not produce significantly better results.

Application

One of the assumptions of the filtering procedure used in the Operation is that the observed snow-water-equivalent values are independent from one observation to the next. As the time period between updates becomes shorter, this assumption becomes less valid. The methodology has only been tested with a monthly updating frequency. More frequent updating should be tested before it is used operationally. The filtering procedure could be extended to account for correlation between observations if more frequent updating is required. Currently, the only safeguard available is that the user is prevented from updating the states at the beginning of an execution since an update may already have taken place on this date.

References

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