II.3-API-CONT CONTINUOUS API MODEL

Introduction

Antecedent Precipitation Index (API) based procedures have been used for many years by River Forecast Centers (RFCs) for producing flood forecasts (Linsley et al, 1949). The API procedures developed by the RFCs are applied on a storm basis. The API value at the beginning of the storm is typically related to time of the year, storm duration and storm rainfall to compute storm runoff (see Figure 1a). Incremental runoff is computed by subtracting the total storm runoff at the end of a period from that at the beginning. A unit hydrograph is then applied to the incremental runoff values to produce a discharge hydrograph. Since storm or event API procedures only compute surface or storm runoff, baseflow needs to be added to the surface runoff hydrograph to produce the total discharge. For short-term river or flood forecasts (hours or days into the future) satisfactory estimates of baseflow can usually be determined. During floods, errors in baseflow estimates have a minimal impact. During recessions it is relatively easy to make a several day projection of discharge. Many RFCs have continued to use API based rainfall-runoff models because they are simple to understand, easy to update when observed values differ from computed estimates and generally do a good job forecasting floods when properly applied.

In recent years two problems have arisen related to the use of API based rainfall-runoff models by the RFCs.

First, the need for water management forecasts is increasing dramatically. For water management purposes predictions are often needed for weeks or months into the future, plus in many cases low flow values are of interest. Within NWSRFS, the Extended Streamflow Prediction (ESP) System is used to generate such forecasts. For general ESP purposes, a model must be able to accurately simulate all flow levels for extended periods. Event API models cannot do this, thus the RFC is faced with switching to a different type of model or using one model for flood forecasting and another for ESP applications. Neither of these options is appealing to some of the RFCs.

Second, it is very difficult to calibrate event API models in conjunction with other hydrologic models for a watershed. The data used for calibration of an event API model typically includes the API value at the beginning of the event and the storm rainfall, both computed from precipitation data; the date and duration of the storm; and the total surface runoff for the event. Total runoff is computed by separating surface runoff from baseflow using one of the standard techniques for baseflow separation. The calibration is then done by either manually deriving the coaxial graphical relationship between the variables (see Figure 1a) and/or using computer techniques to minimize the error between computed and observed storm runoff. A unit hydrograph to be used in conjunction with an event API model can be derived based on events when all surface runoff is generated in a single time period or when uniform runoff can be assumed for several periods. Unit hydrographs derived from other storms are not directly compatible with the event API model because the distribution of runoff used to derive the unit hydrograph is not the same as would be produced by the API model if it was applied to the same event. For other hydrologic models, such as a snowmelt model, that might be used in conjunction with an API model, the output variables of the model rarely can be isolated by analyzing a hydrograph. Thus, these other models cannot be calibrated for the watershed. Currently when a snow model is used with an event API model by the RFCs, the snow model parameters are based on calibrations done somewhere in the area in conjunction with a conceptual rainfall-runoff model or on point calibrations of the snow model using observed water-equivalent data. The calibration procedures provided within NWSRFS generally cannot be used for an event API model.

Since several RFCs prefer to use an API based rainfall-runoff model for flood forecasting, the Continuous API Model was developed so that an API based model that could be used with the ESP and calibration systems would be available within NWSRFS. The Continuous API Model computes runoff on an incremental, not on a storm, basis and generates both surface and baseflow runoff amounts.

Background

In the 1960's a continuous API-type model was developed for use within the Office of Hydrology in order to compare emerging conceptual models with API based rainfall-runoff procedures (Sittner et al, 1969). The original model has since been revised to simplify some of the equations and reduce the number of parameters (Nemec and Sittner, 1982). This continuous API model was developed on the premise that an event API rainfall-runoff relationship could be converted to an incremental relationship by replacing the duration quadrant with a retention index (RI) quadrant. RI reflects whether surface conditions are dry (typical state at the beginning of an event) or wet (condition during an event when interception, depression and upper zone moisture storages have been satisfied). The difference between surface runoff computations in an event API model and Sittner's continuous model can be described by using Figure 1b. An event API model basically uses one curve for the entire event based on antecedent conditions at the beginning of the event. Sittner's model moves from curve to curve as the API and retention index change during the event (typically moves to curves reflecting wetter conditions as the event progresses). However, incremental rather than storm precipitation is used to enter the relationship so that only the beginning portion of the curves, where the most curvature exists, are typically used. It is not clear whether this curvature should exist for every time period during an event. This creates some doubt as to whether the same precipitation-runoff relationship or even the same form of equations, should be used for an incremental API-type model as is used for an event model.

Rather than using the Sittner continuous API model the model this model was developed because of:

- o Doubt that the same equational form of the precipitation-runoff relationship can be used in both an incremental and event API model.

- o The belief that some of the model components could be simplified and thus required fewer parameters and be more easily visualized by the user.

- o Personal preference.

Description of the Model

The Continuous API Model consists of 4 quadrants (see Figure 2), equations to compute baseflow runoff and a few additional features including an option to account for the effect of frozen ground on runoff. The four quadrants perform the following functions:

- o The first quadrant accounts for the seasonal relationship between API and current soil-moisture conditions,

- o The second quadrant accounts for surface moisture conditions,

- o The third quadrant computes the incremental surface runoff based on surface and overall soil-moisture conditions and

- o The fourth quadrant computes what portion of the precipitation that does not become surface runoff enters groundwater storage.

Baseflow runoff is computed based on the total water in groundwater storage and the amount that has entered the storage in the recent past. The model also allows for impervious area runoff and riparian vegetation losses.

1st quadrant

The first quadrant serves the same function in all API based models. This quadrant accounts for the seasonal variations between API and an index to soil-moisture conditions. The index is usually referred to as the Antecedent Index (AI).

Computation of AI: The equations used to compute AI from API are basically the same as used in the West Gulf RFC API model (McCallister, 1963) and are the same as currently used by Sittner for his continuous API model. The only differences are: 1) the variation between wet and dry curves is represented differently and 2) optional ways to express the seasonal variation are included. The equations are:

$$AI_w = AIXW*CW^{API} \tag{1}$$

$$AI_d = AIXD*CD^{API} \tag{2}$$

$$AI = AI_w + y*(AI_d - AI_w) \tag{3}$$

where AI is the Antecedent Index (inches)
AIXW is the intercept of wet curve (i.e. AI value when API = O and y = O) (inches)
CW is the wet curve curvature constant (0.0 < CW < 1.0)
AIXD is the intercept of dry curve (i.e. AI value when API = O and y = 1) (inches)
CD is the dry curve curvature constant (0.0 < CD < 1.0)
y is the fractional distance between wet and dry conditions (y = 0 is wet, y = 1 is dry)

Computation of API: The equations use to compute API are:

$$API_2 = K_p * API_1 + P \tag{4}$$

$$K_p = APIK^{(\Delta t/24)} \tag{5}$$

$$API_2 \le APIX \tag{6}$$

where API is the Antecedent Precipitation Index (inches), (subscripts refer to beginning and end of the time period)
P is the precipitation or rain + melt (inches)
APIK is the daily API recession rate (0 < APIK < 1.0- normally assumed to be 0.9)
Δt is the length of the time period (hours)
K_p is the API recession rate for the time period
APIX is the Maximum value that API can attain.

An upper limit is provided for API because with sufficient rainfall or rain + melt, the soil will become saturated and any additional water goes to runoff, not to increasing the level of soil saturation. The maximum API value is only attained during major flood events.

When a snow cover exists, the API recession rate may need to be reduced. The reduction in API and the subsequent increase in AI is due to both evaporation from the soil and drainage of water in excess of field capacity. AI is an index to the total wetness of the soil. Since a snow cover will inhibit evaporation, the APIK recession rate should be reduced. The amount of reduction is a function of the typical climate conditions of the basin to which the model is being applied. For example, in the upper Midwest a snow cover may exist for most of the winter. If fall soil-moisture conditions are to influence spring runoff, the API recession rate must be reduced to 1.0 or nearly that amount to retain a memory of fall conditions. On the other hand, in a more temperate area where periodic rain or melt periods may occur when a snow cover exists, the snow cover may reduce evaporation, but does not affect water draining through the soil. In such an area, a much smaller reduction in the API recession rate is warranted when snow exists. If the API recession rate were set to 1.0, very large API values could build up during rain-on-snow or melt periods and cause even small amounts of rain + melt to produce a large percent surface runoff.

When the areal extent of the snow cover is known, APIK becomes:

$$APIK_s = APIK + (APIKS-APIK)*S_c \tag{7}$$

where $APIK_s$ is the daily API recession rate when snow exists
S_c is the areal extent of snow cover (decimal fraction)
APIKS is the daily API recession rate with 100 percent snow cover ($APIK \leq APIKS \leq 1.0$)

When only the water-equivalent and not the areal extent of the snow cover is known, S_c is set to 1.0 whenever the water-equivalent exceeds 0.1 inches ($S_c = 0.0$ otherwise).

Seasonal variation: The most common method to account for seasonal variation is to use time of the year. Time of the year is specified by week number which is computed as:

$$W_n = D_j/7.0 \tag{8}$$

where W_n is the week number
D_j is the Julian day (January 1 = 1, December 31 = 365; February 29 and March 1 have same value)

In order to use week number in the 1st quadrant, the week numbers when the wettest and driest conditions typically exist need to be specified. The seasonal variation between wet and dry conditions is expressed as:

$$y = \left[0.5 + \frac{\cos[\pi * (1-f)]}{2}\right]^{CS} \tag{9}$$

where f is the fractional distance between WKD and WKW (f = 1 when W_n= WKD and f = 0 when W_n = WKW)
WKW is the week number when the wettest conditions typically exist
WKD is the week number when the driest conditions typically exist
CS is the seasonal curvature exponent (CS > 0.0)

When going from wet to dry conditions (typically early spring to late summer), the value of CS is fixed at 1.0, thus resulting in a sinusoidal variation. When going from dry to wet conditions, the parameter CS controls the shape of the seasonal variation. A value of CS considerably greater than 1.0 is typically needed in areas where there is a rapid transition from dry to wet conditions in the fall. Rapid transitions from dry to wet conditions occur in basins where a large soil moisture deficit typically develops over the summer due to evapotranspiration significantly exceeding rainfall and the deficit is reduced to zero over a relatively short period in the fall due to increased rainfall and decreased evapotranspiration. In areas where the trees lose their leaves over a relatively short period in the fall, the decrease in evapotranspiration is accentuated. The seasonal variation in y is shown in Figure 3.

One alternative method of accounting for the seasonal variation is the use of an Antecedent Evaporation Index (AEI). This method has been used by the Middle Atlantic RFC. AEI is computed on a daily basis as:

$$AEI_2 = AEIK * AEI_1 + PE \tag{10}$$

where AEI is the Antecedent Evaporation Index (inches)
AEIK is the daily AEI recession rate ($0.0 < AEIK < 1.0$)
PE is the daily potential evaporation or ET-demand (inches); when snow exists PE is adjusted using Equation 18

The minimum AEI value typically occurs a month or so after the time when minimum PE values occur and the maximum AEI a month or so after when maximum PE values occur. When using AEI, the seasonal variation is expressed as:

$$y = \frac{AEI-AEIN}{AEIX-AEIN} \tag{11}$$

where AEIN is the minimum allowed AEI value (inches)-corresponds to wettest time of year (if AEI < AEIN, AEI = AEIN)
AEIX is the maximum allowed AEI value (inches) - corresponds to driest time of year (if AEI > AEIX, AEI = AEIX).

The sinusoidal variation explicitly built into Equation 9 (CS = 1.0) occurs naturally in Equation 11 because PE and thus AEI exhibits a sinusoidal pattern.

The second alternative method of accounting for the seasonal variation is through the use of an Antecedent Temperature Index (ATI). The ATI is a weighted mean temperature and is computed as:

$$ATI_2 = ATI_1 + ATIR*(Tm-ATI_1) \tag{12}$$

where ATI is the Antecedent Temperature Index (DEGF)
ATIR is the temperature weighing factor ($0.0 < ATIR \leq 1.0$)
Tm is the mean daily air temperature (DEGF)

When using ATI the seasonal variation is expressed as:

$$y = \frac{ATI-ATIN}{ATIX-ATIN} \tag{13}$$

where ATIN is the minimum allowed ATI value (DEGF) - corresponds to wettest time of year (if ATI < ATIN, ATI = ATIN)
ATIX is the maximum allowed ATI value (DEGF) - corresponds to driest time of year (if ATI > ATIX, ATI = ATIX)

Similarly to AEI, ATI naturally exhibits a sinusoidal variation.

The possible advantage of using AEI or ATI rather than week number is to better account for abnormal conditions. AEI or ATI should indicate a abnormally cold spring or an abnormally warm fall which should cause a shift in the API vs AI relationship for that time of year.

2nd quadrant

The 2nd quadrant of the Continuous API Model adjusts the AI value computed in the 1st quadrant for the effect of surface moisture. The result is an adjusted or final AI value (AI_f). It is assumed that when surface moisture conditions are wet that AI_f = AI. When surface moisture is dry, AI_f needs to be increased. This causes surface runoff to be decreased in the 3rd quadrant to reflect interception, depression storage and upper zone moisture losses that occur when the surface is dry. The 2nd quadrant accounts for the initial abstraction loss that occurs at the beginning of an event.

Computation of AI_f: The form of the equation used to compute AI_f as a function of AI and the surface moisture conditions is similar to the form of the equations used in the 1st quadrant except that the curvature constant is fixed. The equation is expressed as:

$$\frac{SMI}{SMIX} = 0.9^{\left[\frac{AI_f}{AI} - 1\right]} \qquad (14)$$

where SMI is the Surface Moisture Index (inches)
SMIX is the maximum value of SMI (inches)
AI_f is the Final Antecedent Index (inches)

When the ratio SMI/SMIX = 1, the surface is saturated (i.e. interception, depression and surface moisture storages are full).

Solving Equation 14 for AI_f gives:

$$AI_f = AI * \left[\frac{\ln\left(\frac{SMI}{SMIX}\right)}{\ln(0.9)} + 1\right] \qquad (15)$$

Equation 15 plots as a straight line for each SMI/SMIX ratio (see Figure 2) The constant 0.9 causes the 2nd quadrant to act almost as a threshold storage (i.e. very little surface runoff can be generated until SMI = SMIX).

Computation of SMI: Surface moisture conditions dry out much faster in the summer than in the winter because evaporation rates are much higher in the summer. This needs to be reflected in the computation of SMI. SMI is computed by the equation:

$$SMI_2 = SMI_1 - E*(SMI_1/SMIX) + P \qquad (16)$$

where E is the evaporation (inches)

If the AEI option is used in the first quadrant, then actual evaporation values are used in Equation 16. When week number or ATI are used to express seasonal variation, daily evaporation is computed as:

$$E_d = 0.5*(PEX+PEN) + 0.5*(PEX-PEN) *\sin\left[\frac{2*\pi*(D_j-105)}{365}\right] \qquad (17)$$

where E_d is the daily evaporation estimate (inches)
PEX is the maximum daily evaporation rate, assumed to occur on July 15th (inches)
PEN is the minimum daily evaporation rate, assumed to occur on January 15th (inches)

E is obtained from E_d by assuming uniform evaporation during the day. Values for PEX and PEN can be obtained from an evaporation atlas such as those produced by Farnsworth et al, 1982. In some cases the values derived from the atlas should be adjusted for the effect of vegetation (e.g. January values should be reduced in areas with deciduous forests or cold climates). When a snow cover exists, the evaporation is reduced by:

$$E_s = E*(1-S_c) + E*S_c*EFC \tag{18}$$

where E_s is the evaporation when a snow cover exists (inches)
EFC is the effective forest cover (decimal fraction)

The effective forest cover can be estimated by taking the portion of the area covered by conifers times the average canopy density.

When actual evaporation data are used (i.e., AEI defines seasonal variation), the values in Equation 17 take on a different meaning. PEX and PEN become adjustment factors for July and January 15th and E_d becomes the vegetation adjustment for the current day.

It should be noted that Equation 16 has the same form as the equation used to compute upper zone tension water contents in the Sacramento soil-moisture accounting model (Burnash et al, 1973). Also Equation 16 gives the same results as if it was written in the form of Equation 4 with the daily recession rate equal to 1.0 minus (E_d /SMIX). Thus, SMI could be calculated in the same way that API is computed only with a seasonally varying recession rate. The form of Equation 16 and the use of evaporation data makes it easier for the user to estimate the parameter values.

3rd Quadrant

The 3rd quadrant of the Continuous API Model computes surface runoff knowing AI_f and the amount of precipitation. Earlier it was indicated that some doubt exists as to whether the same relationship can be used for this quadrant in a continuous API model as is used in an event model. Also the typical equations used for this quadrant in previous API models involve 4 or 5 parameters which are not easy to visualize. Thus in this model a much simpler approach was taken.

Computation of Surface Runoff: The model assumes that the fraction of the precipitation that becomes surface runoff increases as AI_f decreases and reaches a maximum when AI_f = 0. This is expressed as:

$$F_s = FRSX*0.7^{AI_f} \tag{19}$$

where F_s is the fraction of precipitation that becomes surface runoff (decimal fraction)

FRSX is the maximum percent runoff (decimal fraction)

The curvature constant in Equation 19 has been fixed at 0.7. Because of the common functional form of equations in the model (Equations 1, 2, 19 and 23) and because the second quadrant is linear, a curvature parameter is not needed in Equation 19. If a different curvature constant is selected, new values of AIXW, AIXD, AICR, CG and RVAI can be computed such that the model will produce exactly the same results. The value of 0.7 was selected so that F_s does not vary too quickly or too slowly as a function of AI_f.

The amount of surface runoff is then computed as:

$$R_s = F_s * P \tag{20}$$

where R_s is the surface runoff (inches)

Some would think that the maximum fraction of surface runoff should be 1.0 (i.e. F_s should equal 1.0 when $AI_f = 0.0$). While this is the case for many watersheds there are also many watersheds that never reach 100 percent surface runoff. Watersheds with high saturated soil permeability never reach 100 percent runoff even near the end of a very large event. For example, at the French Broad River at Rosman, North Carolina from September 28 to 30, 1964 the remnants of a hurricane dropped over 12 inches of rain on the watershed. An additional 1.5 inches occurred over the next 3 days. On October 4th and 5th, 9.8 inches of rain from another hurricane produced the flood of record. The percent surface runoff for this record event was only 32 percent. For this reason the parameter FRSX is needed.

4th Quadrant

The 4th quadrant is used to compute what portion of the precipitation that does not become surface runoff (i.e. $P-R_s$) enters groundwater storage and eventually becomes baseflow runoff. The water that does not become surface runoff or groundwater inflow enters soil-moisture storage or becomes recharge to deep aquifers. No accounting of this water is made in an API model.

<u>Computation of Groundwater Inflow</u>: Based on soil-moisture conditions either none, some or all of the $P-R_s$ quantity enters groundwater storage. It is first assumed that when SMI is less than SMIX (i.e. surface storages not full), that groundwater inflow (G_i) is zero. Second it is assumed that when SMI = SMIX and the soil is wet enough, that $G_i = P-R_s$. Since AI_f is the available index to soil conditions, there is a value of AI_f below which all of the remaining water enters groundwater storage. This value is referred to as the critical AI_f value (AICR). When AI_f is greater than AICR, the fraction of $P-R_s$ entering groundwater storage is reduced and approaches zero as AI_f approaches infinity (see Figure 2). The equations used to compute G_i are:

When SMI < SMIX:

$$F_g = 0.0 \tag{21}$$

where F_g is the fraction of $P-R_s$ that enters groundwater storage (decimal fraction)

When SMI = SMIX and $AI_f \leq AICR$:

$$F_g = 1.0 \tag{22}$$

where AICR is the critical AI_f value (i.e. AI_f value below which $F_g = 1.0$) (inches)

When SMI = SMIX and $AI_f > AICR$:

$$F_g = CG^{(AI_f - AICR)} \tag{23}$$

where CG is the curvature constant for groundwater inflow (0.0 < CG < 1.0)

The actual amount of groundwater inflow is then:

$$G_i = F_g * (P - R_s) \tag{24}$$

where Gi is the groundwater inflow (inches)

Baseflow Runoff

The baseflow runoff equations of the Stanford Watershed Model (Crawford and Linsley, 1966) are used to compute runoff from groundwater storage in the Continuous API Model. The Stanford Model baseflow component is simple, but yet has proven to adequately represent baseflow runoff in a wide variety of basins. The Stanford and Sacramento Models represent baseflow runoff in a very similar manner. Both models assume that there are two baseflow runoff components. First, there are the aquifers that feed the stream during long periods with no groundwater recharge. In the Sacramento Model this is termed primary baseflow runoff. Second, there are aquifers that drain more rapidly and only feed the stream for weeks or months after a period of recharge. The Sacramento Model refers to this drainage as supplemental baseflow runoff. When both sets of aquifers are contributing, the resulting baseflow recession rate is a weighted average of the individual recession rates for each aquifer.

Baseflow Runoff Computations: A baseflow index (which is analogous to API) is used to indicate the amount of groundwater inflow that has occurred in the recent past. This index is computed as:

$$BFI_2 = K_g * BFI_1 + G_i \tag{25}$$

$$K_g = BFIK^{(\Delta t/24)} \tag{26}$$

where BFI is the Baseflow Index (inches)
K_g is the BFI recession rate for the time period.
BFIK is the daily BFI recession rate

BFIK is similar to the supplemental baseflow recession rate in the Sacramento Model (i.e. BFIK ≈ 1.0 - LZSK where LZSK is the daily lower zone supplemental withdrawal rate in the Sacramento Model).

Baseflow runoff is then computed as:

$$R_g = (1.0-K_b)*(1.0+BFIM*BFI)*G_s \tag{27}$$

$$K_b = BFPK^{(\Delta t/24)} \tag{28}$$

where
- R_g is the baseflow or groundwater runoff (inches)
- K_b is the primary baseflow recession rate for the time period
- BFPK is the daily primary baseflow recession rate
- BFIM is the weighing factor (BFIM $\geq$ 0.0)
- G_s is the groundwater storage contents (inches)

BFPK is the same as the primary baseflow recession rate in the Sacramento Model (i.e. BFPK = 1.0 - LZPK where LZPK is the daily lower zone primary withdrawal rate in the Sacramento Model). BFIM determines the relative magnitude of supplemental versus primary baseflow runoff. If BFIM = 0.0 only primary runoff occurs.

The change in the groundwater storage contents are then computed as:

$$G_{s_2} = G_{s_1} + G_i - R_g \tag{29}$$

Additional Features

The Continuous API Model can account for constant impervious area runoff and riparian vegetation losses. Also the computational method used by the model needs to be described.

Impervious Runoff: The model computes impervious runoff as:

$$R_i = PIMPV*P \tag{30}$$

where
- R_i is the impervious area runoff (inches)
- PIMPV is the fraction of the watershed that acts as an impervious area (decimal fraction)

Riparian Vegetation Loss: When the soil is quite dry, riparian vegetation will withdraw water from groundwater seeping into the stream. The antecedent index (AI) is used to reflect moisture conditions. Riparian losses can occur when AI exceeds a specified value (RVAI). Riparian losses are computed as:

When AI $\leq$ RVAI:

$$L_r = 0.0 \tag{31}$$

where RVAI is the AI value above which riparian vegetation losses can occur (inches)

L_r is the amount of riparian vegetation loss (inches)

When AI > RVAI:

$$L_r = RIVA*E*\left(1.0-\frac{SMI}{SMIX}\right)*\left(\frac{AI-RVAI}{AIX-RVAI}\right) \quad (32)$$

where RIVA is the fraction of the watershed covered by riparian vegetation (decimal fraction)
AIX is the maximum AI value for the current day [AIX = AIXW + y*(AIXD-AIXW)] (inches)

The term E*(1.0-SMI/SMIX) represents the residual evaporation demand from the surface layers and ((AI-RVAI)/(AIX-RVAI)) crudely represents that the residual evaporation demand from the rest of the soil increases as AI increases.

Computation Method: The Continuous API Model uses an explicit solution to the equations where the value of the variables at the start of the time interval represent the conditions during the interval. To avoid significant errors during periods with large amounts of precipitation, no time interval is allowed to contain more than 0.2 inches of precipitation. Thus, during periods with large amounts of precipitation, the model increments along the curves by subdividing the period into shorter intervals.

Total Runoff: The total runoff generated by the Continuous API Model is computed as:

$$R = R_i + (R_s+R_g)*(1-PIMPV) - L_r \quad (33)$$

where R is the total runoff (inches)

Frozen Ground Effects

Frozen ground can have a significant effect on the amount of runoff that results from rain or snowmelt. When the ground freezes, the water that is in the soil pores will freeze causing a blockage and thus a reduction in the infiltration rate. Unless the soil is quite saturated when freezing occurs, there is initially very little reduction in the rate of infiltration. This is because there is not enough water in the pore spaces, especially in the larger spaces. The infiltration rate is not reduced significantly until enough rain or snowmelt enters the soil and freezes to restrict the entry of water into the ground. As thawing occurs, the water frozen in the pores melts allowing for water to again infiltrate at a rate unrestricted by ice. The Continuous API Model attempts to account for the effect of frozen ground by using a frost index and a frost efficiency index. The frost index indicates the extent of frost in the soil (Anderson and Neuman, 1984). The frost efficiency index indicates the degree to which the soil pores have been filled with ice.

The frost index and frost efficiency index are intended to be representative of the portion of the basin that can exhibit significant frozen ground effects on runoff. In general, open agricultural areas experience much more significant frozen ground effects than conifer forests with a thick litter layer. If a watershed contains a mixture of open and forested areas, the indices should be used to estimate frozen ground conditions in the open areas. The overall effect of frozen ground on runoff for the basin is computed using an effective frost area parameter.

Frost Index

The empirical frost index is computed as:

$$FI_2 = FI_1 + \Delta FI \tag{34}$$

where FI is the frost index (DEGF), (subscripts refer to the beginning and end of the time period)

FI is always $\leq$ 32 DEGF. The change in FI is computed differently depending on whether the air temperature is above or below freezing. When the air temperature is below freezing (i.e. frost is typically growing):

$$\Delta FI = -C*\sqrt{(T_a-32)^2+(FI_1-32)^2} - C*(FI_1-32) + GHC*(24/\Delta t) \tag{35}$$

where C is the frost coefficient for the time interval
T_a is the air temperature (DEGF)
GHC is the daily thaw rate due to ground heat (DEGF)

When the air temperature is above freezing (i.e. the frost is thawing):

$$\Delta FI = C*(T_a-32) + GHC*(24/\Delta t) \tag{36}$$

Thawing can also occur due to heat transfer from rainwater, however, since the amount of heat transferred is generally much less than from the atmosphere, this factor is neglected.

Figure 4 depicts the change in the frost index when GHC is zero. The frost index grows most rapidly when the air temperature is considerably below the current FI value. The frost index will continue to grow whenever the temperature is below freezing unless the change due to temperature is less than GHC.

The frost coefficient that primarily controls the change in the frost index is dependant on the heat transfer characteristics of the upper soil layers. Frost will develop faster in an area with bare ground than in an area covered by litter. When snow is on the ground, the frost coefficient needs to be reduced due to the insulating effect of the snow cover. The frost coefficient is computed as:

$$C = CSOIL*(\Delta t/6)*(1-S_C) + CSOIL*(\Delta t/6)*S_c*(1-CSNOW)^{W_e} \tag{37}$$

where CSOIL is the frost coefficient for non-snow covered soil (6 HR^{-1})
CSNOW is the reduction in CSOIL per inch of snow water-

equivalent (decimal fraction)
W_e is the snow water-equivalent (inches)

Figure 5 shows the reduction in the frost coefficient as a function of snow water-equivalent for typical values of the CSNOW parameter.

Frost Efficiency Index

The frost efficiency index varies between zero (frost has no effect on runoff) and 1.0 (concrete frost exists over the entire area, thus there is a 100 percent runoff rate). When there is insufficient frost in the soil to freeze the water in the pores spaces, the frost efficiency index becomes zero.

The frost efficiency index is computed as:

$$FEI_2 = FEI_1 + \Delta FEI \tag{38}$$

where FEI is the frost efficiency index (subscripts refer to the beginning and end of the time period)

Frozen ground has no effect on runoff when FI is greater than or equal to a specified value, thus:

When FI $\geq$ FICR:

$$FEI = 0.0 \tag{39}$$

where FICR is the critical frost index (DEGF)

FEI can change due to water within the soil freezing when the frost index grows, rain or melt water freezing when it enters frozen soil and thawing when the air temperature is above freezing.

The change in FEI due to water in the soil freezing as frost develops is computed as:

When FI $\geq$ FICR or $FI_2 > FI_1$:

$$\Delta FEI_f = 0.0 \tag{40}$$

When FI < FICR and $FI_2 < FI_1$:

$$\Delta FEI_f = (1-FEI_1)*CF*(1-AI_r)^2*(FI_1-FI_2) \tag{41}$$

where ΔFEI_f is the change in FEI due to freezing
CF is the FEI freezing coefficient ($DEGF^{-1}$)
AI_r is the ratio of current AI to AIX, the maximum value for the date

The value of AI_r is an indicator to the soil-moisture conditions. Equation 40 indicates that the soil must be quite wet (i.e. most pore spaces filled with water) before freezing temperatures will significantly reduce infiltration and that the rate of water freezing is reduced as more pores are filled with ice. The insulating effect of a snow cover is built into the computation of the FI values.

The change in FEI due to rain or meltwater freezing when it enters frozen soil is computed as:

When $FI \geq FICR$ or $P = 0.0$:

$$\Delta FEI_p = 0.0 \tag{42}$$

When $FI < FICR$ and $P > 0.0$:

$$\Delta FEI_p = \frac{1}{CP} * R * (1 - AI_r) * P \tag{43}$$

$$R = 0.5 + \frac{COS[\pi * (1 - FI_r)]}{2} \tag{44}$$

where ΔFEI_p is the change in FEI due to precipitation freezing in the soil
CP is the FEI precipitation coefficient (inches)
FI_r is the frost index ratio ((FICR-FI)/70.0); when (FICR-FI) > 70, $FI_r = 1.0$

CP is the amount of precipitation needed to raise FEI from 0.0 to 1.0 with wet soil and maximum frost conditions. Equation 43 indicates that more of the precipitation will freeze and clog the pores when the soil is wet (precipitation is held in the soil and allowed to freeze) and when there is significant frost (more chance of freezing before water percolates below the frost level).

The change in FEI due to thawing is computed as:

When $T_a \leq 32$:

$$\Delta FEI_t = 0.0 \tag{45}$$

When $T_a > 32$:

$$\Delta FEI_t = -C_t * (T_a - 32) \tag{46}$$

$$C_t = CT * (\Delta t/6) * (1 - S_c) + CT * (\Delta t/6) * S_c * (1 - CSNOW)^{W_e} \tag{47}$$

where ΔFEI_t is the change in FEI due to thawing
CT is the FEI thaw coefficient for non-snow covered ground ($DEGF^{-1} * 6HR^{-1}$)

Just as with the frost coefficient used in FI computations, the FEI thaw coefficient is reduced when snow covers the ground.

The total change in the frost efficiency index is:

$$\Delta FEI = \Delta FEI_f + \Delta FEI_p + \Delta FEI_t \tag{48}$$

The frost efficiency index is used to compute the additional surface runoff that occurs due to ice filled soil pores. It is assumed that the rate of increase of additional surface runoff increases as FEI increases (i.e. the relationship between FEI and additional surface runoff is not linear). The frost efficiency, as noted earlier, is only applied to the portion of the watershed where frozen ground has a significant effect on runoff. The fraction of surface runoff when frozen ground exists is computed as:

$$F_s' = F_s + (1-F_s)*FEI^2*EFA \tag{49}$$

where F_s' is the fraction of precipitation that becomes surface runoff when frozen ground exists (decimal fraction
EFA is the effective frost area (decimal fraction)

F_s' is then used in place of F_s in Equation 20. Figure 6 shows what the 3rd quadrant of the model looks like for an effective frost area of 1.0.

Additional Frozen Ground Modifications

In addition to modifying the fraction of the precipitation that becomes surface runoff, the computation of API and SMI need to be modified when frozen ground exists. The API recession rate is assumed to be 1.0 and the evaporation amount used to compute SMI is assumed to be zero over the EFA when frozen ground is present. When significant frozen ground exists (FI < FICR), the API recession rate becomes:

$$APIK_f = 1.0*EFA + (1-EFA)*APIK_s \tag{50}$$

where $APIK_f$ = daily API recession rate when frozen ground exists

When frozen ground exists, the evaporation value used in SMI computations (Equation 16) becomes:

$$E_f = (1.0-EFA)*E_s \tag{51}$$

where E_f is the evaporation when frozen ground exists (inches)

In addition to reducing the API recession rate, the precipitation value used in computing API (Equation 4) is modified when frozen ground exists. As the soil pores fill with ice (i.e. FEI increases), more of the subsequent precipitation becomes surface runoff and less goes to increasing soil moisture. Thus, the API value should not increase by the full amount of new precipitation. If no reduction is applied to the precipitation amount used to compute the change in API, the soil will be too wet after the frost is gone. Thus, when frozen ground exists, Equation 4 becomes:

$$API_2 = K_p*API_1 + (1-(FEI*EFA))*P \tag{52}$$

The recession rate, K_p, is computed from $APIK_f$ using Equation 5.

<u>Parameter Summary</u>

The parameters of the basic Continuous API Model and the parameters associated with the frozen ground option are summarized in this section.

Basic Model Parameters

The parameters of the basic Continuous API Model can be divided into 3 categories.

Category 1: This category contains the parameters that need to be determined through trial-an-error and automatic calibration procedures. These parameters typically can't be determined based on a hydrograph analysis or physiographic factors. The category 1 parameters are:

- AIXW, CW, AIXD, CD - These 1st quadrant parameters define the wet and dry curves relating API to AI and are probably the most critical parameters to be determined by the calibration process.

- CS - Seasonal curvature exponent used to control the transition from dry to wet curves in the 1st quadrant. A value of 1.0 results in a sinusoidal transition from late summer to winter conditions. Values considerably greater than 1.0 (e.g. 2.5-4) cause a rapid change in the fall and are indicative of watersheds where the soil-moisture deficit built-up during the summer is reduced to zero over a relatively short period of a month or so in the fall.

- AICR and CG - These 4th quadrant parameters control how much of the precipitation that does not become surface runoff enters groundwater storage. The parameters control the magnitude of baseflow. Changes to 1st quadrant parameters will affect groundwater inflow because the AI values will change, however, changes to AICR and CG will not affect surface runoff computations.

- BFIM - This weighing factor controls the magnitude of faster responding or supplemental baseflow relative to slower responding or primary baseflow. BFIM thus controls the timing of baseflow runoff assuming the two recession rates are reasonably correct.

Category 2: This category contains the parameters that can generally be derived from a hydrograph analysis or from physiographic information about the watersheds. These parameters should require little if any adjustment as part of the calibration process.

- WKW and WKD - The week number of the wettest and driest times of the year can usually be obtained from a general knowledge of the area. WKW generally occurs from late February through early May with the later dates being associated with northern or mountain basins with considerable snowmelt runoff. WKD usually occurs in August or early September.

- APIKS - The daily API recession rate when the ground is completely covered by snow. In areas with long periods of snow cover with little rain or melt, use an APIKS of 1.0 so that soil moisture conditions prior to the snow cover are remembered when snowmelt occurs in the spring. In temperate zones where significant rain or melt can occur when a snow cover exists, APIKS should only be slightly greater than APIK.

- SMIX - The maximum value of the surface moisture index represents, the size of interception, depression and surface moisture storage. In general, significant surface runoff does not occur until SMI = SMIX. The correct general magnitude of this parameter is important, but the results do not appear to be sensitive to small changes in the value of SMIX. A reasonable estimate of SMIX can usually be determined by finding the amount of precipitation needed to cause surface runoff after a dry period in the summer.

- FRSX - This parameter represents the maximum percent surface runoff that can ever occur. In many watersheds a good approximation of FRSX can be derived by computing the percent surface runoff for a very large event that occurs when the soil is wet. The value of FRSX is somewhat greater than the percent runoff for the event since the percent runoff at the end of the event is greater than for the event as a whole. In the case of basins where the maximum percent runoff occurs near the end of extended snowmelt periods, an initial estimate of FRSX is much more difficult to derive.

- PEX and PEN - These are the maximum and minimum daily evaporation rates and are assumed to occur on July 15th and January 15th, respectively. These values are obtained from historical evaporation data. Sometimes the values should be adjusted for the effect of vegetation (e.g. in areas with deciduous forests, the value of PEN should be adjusted downward or even set to zero in northern climates).

- EFC - The effective forest cover is used to adjust evaporation rates when snow exists. It is equal to the fraction of the area covered by conifer forests times the average cover density.

- BFIK and BFPK - These are the daily recession rates for short-term or supplemental baseflow and for long-term or primary baseflow. These values can usually be derived from historical streamflow data.

- PIMPV - The percent impervious area also can usually be estimated from historical data. Streamflow and concurrent precipitation data are required.

- RVAI and RIVA - The values of these riparian vegetation parameters can not be derived in advance, but the presence of riparian losses can be detected. Sharp baseflow recessions during dry summer months indicate that riparian losses exist. Sometimes the flow will go to zero during these periods, but then recover in the fall without the occurrence of significant recharge. When these losses exist, the calibration is normally done with RVAI and RIVA set to zero and then as a final step the riparian loss is included. At this point estimates of RVAI and RIVA can be made by comparing the simulated hydrograph without riparian losses with the observed hydrograph.

<u>Category 3</u>: This category contains the parameters that generally have

the same or a similar value for all watersheds. Very seldom are different values required.

- APIK - The daily API recession is normally set to 0.9.

- APIX - The maximum allowed API value is generally in the range of 8-10 inches.

The parameters for the special seasonal variation options involving AEI and ATI are not included in the parameter summary.

Frozen Ground Parameters

The Continuous API Model parameters involved in frozen ground computations can be divided into those used to compute the frost index and those used in calculating the frost efficiency index and its effect on runoff.

<u>Frost Index</u>: There are 3 parameters used to compute the frost index. These parameters are:

- CSOIL - The frost coefficient for bare ground conditions controls both the growth of the frost index (freezing) and the decay of the frost index (thawing). This is the most important parameter in the calculation of the frost index. Open areas with bare soils should exhibit the greatest amount of frost and the highest CSOIL values, while areas with a litter layer will have less frost and the lower values of the parameter.

- CSNOW - Accounts for the insulating effect of a snow cover. Even a few inches of snow depth can reduce the frost coefficient by 80-90 percent.

- GHC - This parameter controls how the frost index is affected by heat transfer from below the frost layer. Ground heat provides a small, but steady reduction in the frost index. The primary need for GHC is to reproduce the thawing of frozen ground that occurs under a deep snow cover.

<u>Frost Efficiency Index</u>: There are 5 parameters used in computing the frost efficiency index and its effect on runoff. In addition, the CSOIL parameter is also used during FEI calculations. The parameters are:

- FICR - The value of the frost index above which soil frost has no effect on infiltration and the generation of runoff. A small amount of soil frost will essentially have no effect.

- CP - The amount of precipitation that must freeze in order to fill the soil pores with ice. Even when there is deep frost penetration, there will not be much effect on runoff until there is sufficient rain or snowmelt to fill the soil pores and freeze.

- CF - The FEI freezing coefficient controls the increase in FEI during cold periods. Rain or snowmelt does not occur during these periods. The frost efficiency index will increase slowly due to freezing of existing water in the soil pores. CF has a minor effect on the increase in FEI unless there are very high soil-moisture conditions when frost is formed.

- CT - The FEI thaw coefficient controls the decrease in FEI when thawing of the soil occurs. CT will determine how long it will take, once warm weather occurs and the snow melts, for the effect of soil frost on runoff to disappear.

- EFA - The effective frost area controls the portion of the watershed that runoff generation can be significantly affected by frozen ground. The frost index and frost efficiency index values are intended to be representative of this portion of the basin.

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Figure 1a. Typical event API Rainfall-runoff relationship:
graphical coaxial relationship

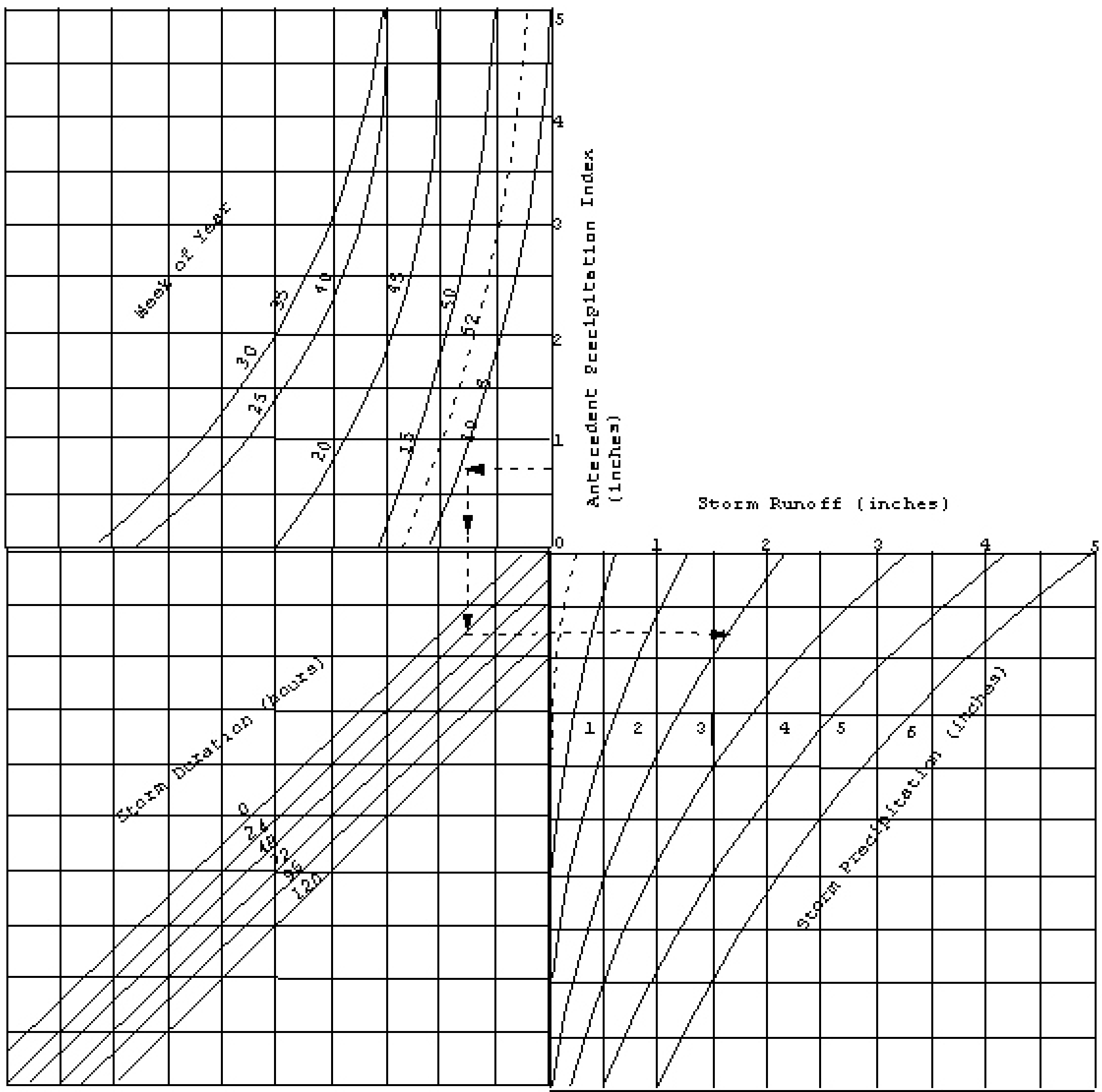

Figure 1b. Typical event API rainfall-runoff relationship: precipitation versus runoff as a function of antecedent conditions

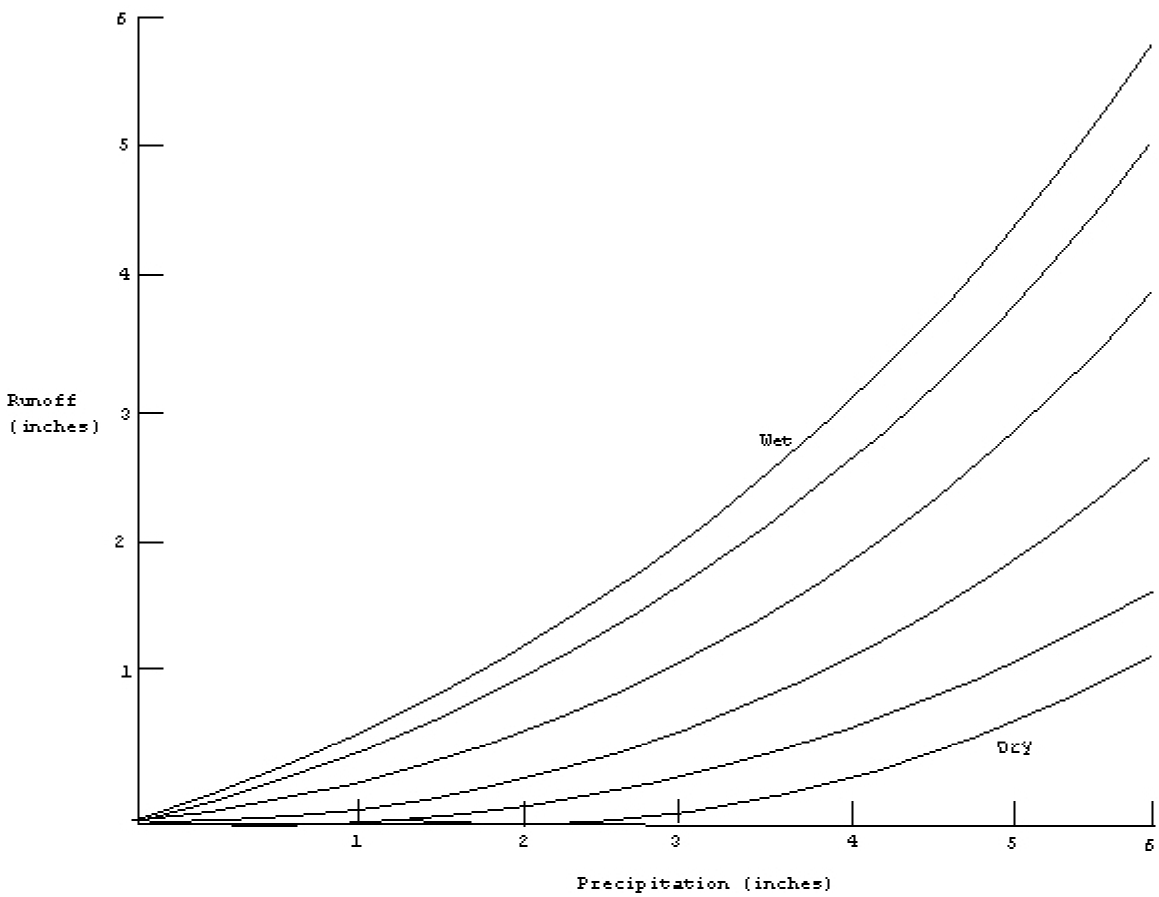

Figure 2. Graph of the 4 quadrants of the Continuous API Model

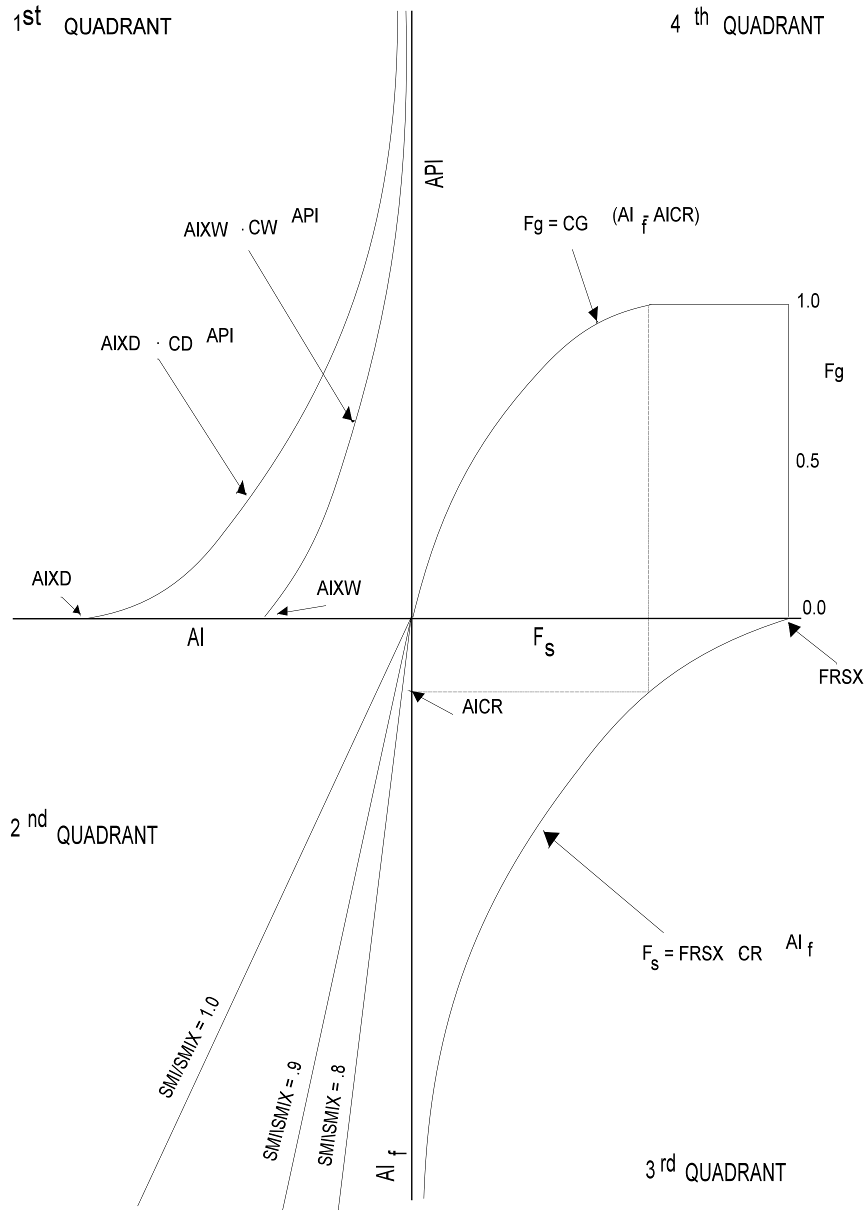

Figure 3. Seasonal variation of y for typical values of WKW and WKD

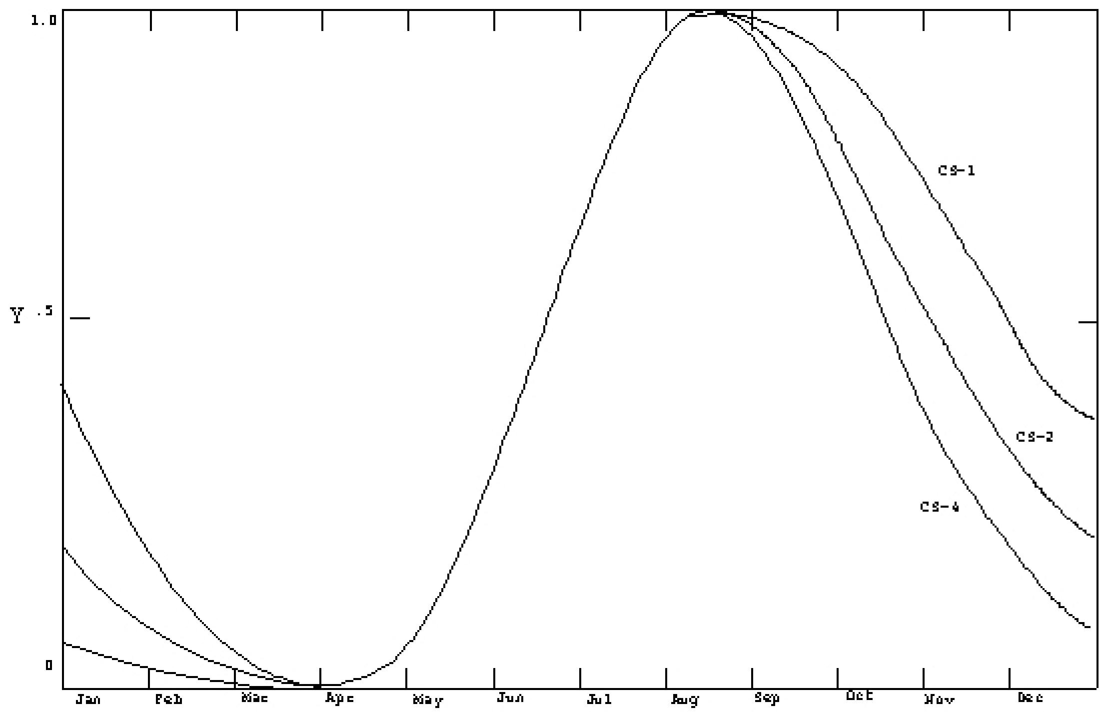

Figure 4. Graphical depiction of the change in the frost index (ΔFI) versus air temperature (T_a) with no ground heat

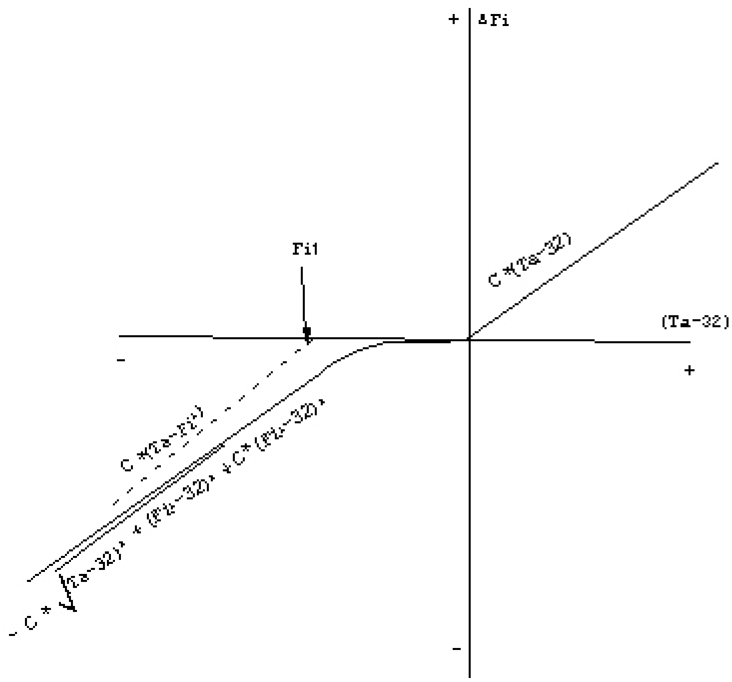

Figure 5. The reduction in the frost coefficient as a function of the amount of snow for typical values of CSNOW

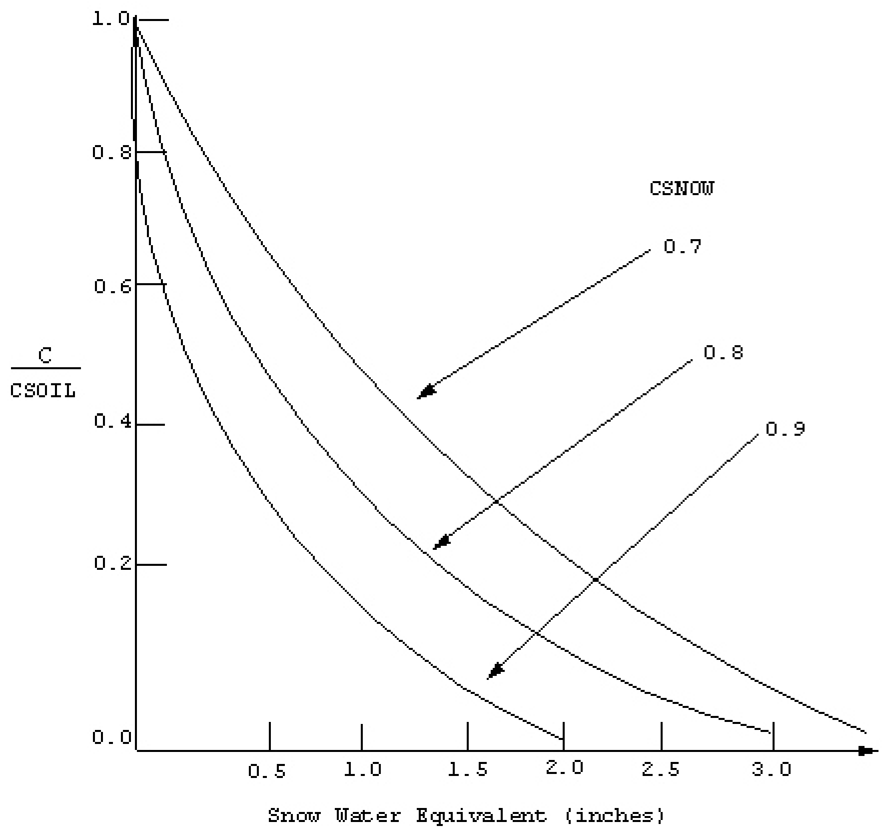

Figure 6. 3rd quadrant of the model when frozen ground is included (assumes an effective frost area of 1.0)