II.4-STAGEQ   STAGE-DISCHARGE CONVERSION FOR A SINGLE VALUED RATING
              CURVE

Introduction

This Section describes the methods available for converting a given
stage or discharge to the associated discharge or stage using
interpolation or extrapolation of a single-valued Rating Curve.

A single-valued Rating Curve is one that has the same stage-discharge
relationship when the river level is rising or falling as illustrated
in Figure 1.

Either linear or logarithmic interpolation can be used to determine
stage from discharge or vice-versa when the given value lies within
the range of the defined Rating Curve points. When the given value
lies outside the specified range then extrapolation is necessary.
Extrapolation can be done using linear, logarithmic or hydraulic
methods to extend the Rating Curve.

Linear Interpolation/Extrapolation

The basic equations used for linear interpolation and extrapolation
are as follows:

1. Determine the slope (a) of the Rating Curve from the two
    points that bracket the given value in the case of
    interpolation or the two points closest to the given value in
    the case of extrapolation. The point with the largest values
    is designated H_u, Q_u and the one with the smallest values is
    H_1, Q_1. The slope is computed as:

$$a = \frac{Q_u - Q_1}{H_u - H_1} \tag{1}$$

2. To convert a given stage (H') to discharge (Q'):

$$Q' = Q_1 + a(H'-H_1) \tag{2}$$

3. To convert a given discharge (Q') to stage (H'):

$$H' = H_1 + \frac{1}{a}(Q'-Q_1) \tag{3}$$

Logarithmic Interpolation/Extrapolation

The basic equations used for logarithmic interpolation and
extrapolation are as follows:

1. Determine the logarithmic slope (a) of the Rating Curve from
   the two points that bracket the given value in the case of
   interpolation or the two points closest to the given value in

the case of extrapolation. The point with the largest values is designated H_u, Q_u and the one with the smallest values is H_1, Q_1. The slope is computed as:

$$a = \frac{\text{Log } Q_u - \text{Log } Q_1}{\text{Log } H_u - \text{Log } H_1} \tag{4}$$

2. To convert a given stage (H') to discharge (Q'):

$$\text{Log } Q' = \text{Log } Q_1 + a(\text{Log } H' - \text{Log } H_1) \tag{5}$$

$$Q' = e^{\text{Log } Q'} \tag{6}$$

3. To convert a given discharge (Q') to stage (H'):

$$\text{Log } H' = \text{Log } H_1 + \frac{1}{a}(\text{Log } Q' - \text{Log } Q_1) \tag{7}$$

$$H' = e^{\text{Log } H'} \tag{8}$$

If Q_1 is equal to zero then Q_1 is set to 0.0001 CMS to allow for logarithmic calculations. In addition stages less than the specified minimum allowable value (H_m) are not used in the computations or generated as the result.

If the Rating Curve does not plot as a straight line on logarithmic paper then the interpolation and extrapolation results will not be correct. When this occurs, an offset value can be used to shift the Rating Curve so that it does plot as a straight line on logarithmic paper. The offset factor (O_f) is subtracted from all the stage values in equations 4, 5 and 7. When an offset factor is used Equation 8 becomes:

$$H' = e^{\text{Log } (H' - O_f)} + O_f \tag{9}$$

In some cases different offset factors need to be applied to different ranges of stage in order to straighten out the logarithmic Rating Curve plot. The stages above which user specified offset factors apply must be defined points on the Rating Curve. Also a given offset factor must be less than the minimum stage to which it applies (H_f) so that the shifted stages will be greater than zero.

Offset factors can also be used to allow negative stages to be used in the logarithmic interpolation/extrapolation equations. In this case negative offset factors insure that positive stage values are used in the equations. If the user does not specify offset factors such that all stages used in the calculations are positive then the program automatically computes a shift factor (S_f) to produce the same results. S_f is used exactly like O_f in equations 4, 5, 7 and 9 except S_f is added to all stage values (i.e. $S_f = -O_f$). The rules used to

determine S_f and the stage below which it is applied (H_s) are as follows:

1. If zero flow is a defined point (H_o, Q_o) on the Rating Curve then:

    a. If specified H_m is less than H_o then set $H_m=H_o$

    b. If $H_o>0.0$ or $H_o=\min(H_f)$ then:

        $S_f=0.0$

        and

        $H_s=H_o$

    c. If $H_o \le 0.0$ and $\min(H_f)>H_o$ then:

        $S_f=0.01-H_o$

        and

        $H_s=\min[\min(H_f),H_p]$

        where H_p is the smallest positive stage value defined

2. If the zero discharge point is not included in the Rating Curve definition then:

    a. If H_m and the stage at the first defined point on the Rating Curve (H_1) are both greater than zero then:

        $S_f=0.0$

        and

        $H_s=H_o$

        where H_o is computed by using $Q_o=0.0001$ CMS

    b. If H_m or $H_1 \le 0.0$ then:

        $S_f=0.01-\min[H_m,H_1]$

        and

        $H_s=\min[\min(H_f),H_p]$

    c. If H_m is less than the computed H_o then:

        $H_m=H_o$

The shift factor will never be applied to the same range of stages to which an offset is applied. The shift factor is also only applied to the portion of the Rating Curve that needs to be shifted to avoid zero or negative values in logarithmic calculations. The shifted

portion of a Rating Curve most likely will not plot as a straight line on logarithmic paper. Therefore it is better off to specify offset factors to insure that reasonable results are obtained during extrapolation at the low end of the Rating Curve rather than to rely on the automatic shift procedure.

Hydraulic Extension

Hydraulic extension is used to determine the stage or discharge associated with a given discharge or stage, respectively, that lies beyond the range of the Rating Curve by examining the hydraulic properties of the stream cross-section.

The Manning equation is the basis of this method:

$$Q = \frac{1}{n} S^{1/2} * A^{5/3}/B^{2/3} = C * A^{5/3}/B^{2/3} \tag{10}$$

> where Q is the discharge
> n is the roughness coefficient
> S is the hydraulic slope
> A is the flow areas below the water surface
> B is the top width at the water surface
> C is:

$$\frac{1}{n} S^{1/2}$$

To determine Q given the stage the program examines the trend of the value C near the Rating Curve end point and extrapolates this value to the stage given. The cross-sectional area and top width are linearly interpolated or extrapolated internally. The discharge associated with the given stage is computed from Equation 10.

If the roughness coefficient (n_2) above the uppermost elevation of the Rating Curve is entered then a composite roughness coefficient will be used for the hydraulic extension and is estimated from:

$$n = \left[\frac{n_1^2 * B_1 + n_2^2 * B_2}{B} \right]^{1/2} \tag{11}$$

> where B   is the interpolated/extrapolated total topwidth, above
>   the uppermost elevation of the Rating Curve, at the
>   water surface elevation
> n   is the composite roughness coefficient; applicable to
>   total topwidth B
> B_1 is the topwidth at the uppermost elevation of the Rating
>   Curve
> n_1 is the roughness coefficient (as determined from
>   extrapolated 'C' values); applicable to topwidth B_1
> B_2 is B - B_1

n_2 is the roughness coefficient above the uppermost
         elevation of the Rating Curve; applicable to topwidth
         B_2

The discharge for the given stage is again computed using Equation
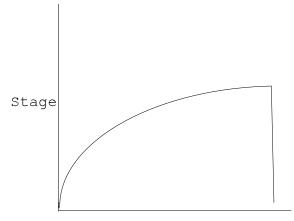10.

To determine stage given discharge, a more complex solution technique
is used as the flow area and top width are both functions of stage
and a solution cannot be explicitly obtained. In this case a
Newton-Raphson iteration technique is employed which uses trial
values for stage in a function that accounts for the difference in
the given discharge and the discharge that would be computed using
the trial stage value. During each iteration a new trial stage value
is computed that reduces the difference between the given discharge
and the trial value discharge. Computation continues until the
change in trial stage values from one iteration to the next is less
that a preset value or until 50 iterations have been made, at which
time a message indicating the occurrence of non-convergence is
printed and the desired stage is found by linear or logarithmic
extrapolation.

Hydraulic extension will be used only at the upper end of the Rating
Curve. Linear or logarithmic extrapolation is used to extend the
Rating Curve at the lower end.

A deficiency was detected in the hydraulic extension option in
dealing with rapid change of channel topwidth. The impact becomes
even more prominent in the case of a sudden presence of a very large
channel topwidth with a very small flow area beneath it. Because the
discharges in the hydraulic extension option are computed from the
hydraulic radius and the area of an equivalent channel, the computed
elevation-discharge relation could be erratic near and above the
elevation where the rapid change of topwidth took place.

Discharges for the higher stages could be less than those for the
lower stages, a result only possible in the loop rating option. A
conveyance approach, which accounts for the channel flow and the
flood plain flow separately, can be used to overcome these problems
but is not currently available.

Figure 1. Stage-Discharge relationship for a single-valued Rating
           Curve