IV.4.2-OPT3-HMLE PROGRAM OPT3 MAXIMUM LIKELIHOOD ESTIMATOR FOR HETEROSCEDASTIC ERROR CASE (HMLE)

Method

The Maximum Likelihood Estimator for the Heteroscedastic Error Case (HMLE) has the following expression:

$$\min_{\theta,\lambda} HMLE = \frac{\sum_{t=1}^{N} w_t * \epsilon_t^2}{N\left[\prod_{t=1}^{N} w_t\right]^{\frac{1}{n}}}$$

where ϵ_t is $OBSQ_t$-$SIMQ_t$ is the model residual at time t, t=1,2,...,N
N is number of data points
OBSQ is the observed flow
SIMQ is the simulated flow
w_t is weight assigned to the data value at time t and is computed as:

$$w_t = f_t^{2(\lambda-1)}$$

where f_t is expected true flow value at time t
λ is unknown transformation parameter which stabilizes the variance

The expected flow value f_t used in OPT3 is approximated by OBSQ.

The HMLE criterion [Sorooshian and Dracup, 1980] is the maximum likelihood, minimum variance, asymptotically unbiased estimator. It is suitable for cases in which the calibration data contain errors that have a normal probability distribution with zero mean and non-stationary variance in time. The variance of the errors is assumed to be related to the level of the output (i.e., the magnitude of flow values). Such errors are believed to be common in streamflow data and can be explained in part by the stage-discharge rating curve. Due to the shape of a rating curve, an error in higher stage readings translates into a larger deviation in the resulting discharge than an error in lower stage readings. Also stages at higher flows are typically more susceptible to measurement errors due to increased turbulence in the river.

The HMLE is basically a weighted DRMS criterion and can be generally used in place of the DRMS criterion. The difference between the HMLE and DRMS criteria is that the former accounts for the stochastic nature of the errors contained in the calibration data while the latter does not. The HMLE criterion should alleviate the drawback in the DRMS criterion that the criterion is heavily dominated by high flow values. There is some evidence that while the calibration period

statistics using HMLE may be somewhat worse

than those obtained using DRMS, the parameter values obtained using HMLE are more robust and provide better results in a verification or forecast mode [Sorooshian, Gupta and Fulton, 1983].

References

Sorooshian, S. and J.A. Dracup, 'Stochastic Parameter Estimation Procedures for Hydrologic Rainfall-Runoff Models: Correlated and Heteroscedastic Error Cases', Water Resources Research, Volume 16(2), pages 430-442, 1980

Sorooshian, S., V.K. Gupta and J.L. Fulton, 'Evaluation of Maximum Likelihood Parameter Estimation Techniques for Conceptual Rainfall-Runoff Models: Influence of Calibration Data Variability and Length on Model Credibility', Water Resources Research, Volume 19(1), pages 251-259, 1983