IV.4.2-OPT3-SCE-UA PROGRAM OPT3 SHUFFLED COMPLEX EVOLUTION (SCE-UA) CONCEPT REPRESENTATION

Search of Two Parameter Space (Duan, 1991; Duan et al., 1992, 1993)

## Method

NGS	number				

- NPG = number of points in each complex
- NPT = number of points in the entire sample population, NPT=NGS\*NPG
- NPS = number of points in each sub-complex

- MAXN = maximum number of trials allowed before optimization is terminated
- ITC = number of shuffling loops in which the criterion value must change by PCNT before optimization is terminated
- PCNT = percentage by which the criterion value must change in ITC shuffling loops
- 1. Generate sample:

Sample NPT points in the feasible parameter space and compute the criterion value at each point. In the absence of prior information, use a uniform probability distribution to generate a sample.

2. Rank points:

Sort the NPT points in order of increasing criterion value so that the first point represents the point with the smallest criterion value and the last point represents the point with the largest criterion value.

3. Partition into complexes:

Partition the NPT points into NGS complexes, each containing NPG points. The complexes are partitioned in such a way that the first complex contains every NGS\*(k-1)+1 ranked point, the second complex contains every NGS\*(k-1)+2 ranked point, and so on, where  $k = 1, 2, \ldots, NPG$ .

Figure 1a shows that a sample population containing NPT (=10)

points is divided into NGS (=2) complexes. Each complex contains NPG (=5) points which are marked by  $\bullet$  and \* respectively. The contour lines in Figures 1 and 2 represent a function surface that has a global optimum located at (4,2) and a local optimum located at (1,2).

4. Evolve each complex:

Evolve each complex independently by taking NSPL evolution steps. Figure 2 illustrates how each evolution step is taken.

In Figure 2, the black dots  $(\bullet)$  indicate the locations of the points in a complex before the evolution step is taken. A subcomplex containing NPS (=3, i.e., forms a triangle in this case) points is selected according to a pre-specified probability distribution to initiate an evolution step. The probability distribution is specified such that the better points have a higher chance of being chosen to form the sub-complex than the worse points. The symbol (\*) represents the new points generated by the evolution steps. There are three types of evolution steps: reflection, contraction and mutation. Figures 2a, 2b and 2d illustrate the 'reflection' step, which is implemented by reflecting the worst point in a sub-complex through the centroid of the other points. Since the reflected point has a lower criterion value than the worst point, the worst point is discarded and replaced by the new point. Thus an evolution step is completed. In Figure 2c, the new point is generated by a 'contraction' step (the new point lies half-way between the worst point and the centroid of the other points), after rejecting a reflection step for not improving the criterion value. In Figure 2e, a 'mutation' step is taken by randomly selecting a point in the feasible parameter space to replace the worst point of the sub-complex. This is done after a reflection step is attempted, but results in a point outside of the feasible parameter space. Another scenario in which a mutation step is taken is when both the reflection step and the contraction step do not improve the criterion value. Figure 2f shows the final complex after NSPL (=5) evolution steps.

Figure 1b shows the locations of the points in the two independently evolved complexes at the end of the first cycle of evolution. It can be seen that one complex (marked by \*) is converging toward the local optimum, while the other (marked by •) is converging toward the global optimum.

5. Shuffle complexes:

Combine the points in the evolved complexes into a single sample population; sort the sample population in order of increasing criterion value; re-partition or shuffle the sample population into NGS complexes according to the procedure specified in Step 3.

Figure 1c displays the new membership of the two evolved complexes after shuffling.

6. Check convergence:

If the number of trials have exceeded MAXN, or the criterion value has not improved by PCNT percent in ITC shuffling loops, stop; else, continue.

7. Check complex number reduction:

If MINGS < NGS, remove the complex with the lowest ranked points; set NGS=NGS-1 and NPT=NGS\*NPG; and return to Step 4. If MINGS=NGS, return to Step 4.

Figure 1d exhibits the two complexes at the end of the second cycle of evolution. It is clear that both complexes are converging to the global optimum at the end of second cycle.

## References

Duan, Q., 'A Global Optimization Strategy for Efficient and Effective Calibration of Hydrologic Models', Ph.D. dissertation, University of Arizona, Tucson, Arizona, 1991

Duan, Q., V.K. Gupta, and S. Sorooshian, 'A Shuffled Complex Evolution Approach for Effective and Efficient Global Minimization', Journal of Optimization Theory and Its Applications, Vol 61(3), 1993

Duan, Q., S. Sorooshian, and V.K. Gupta, 'Effective and Efficient Global Optimization for Conceptual Rainfall-Runoff Model', to appear in Water Resources Research, 1992

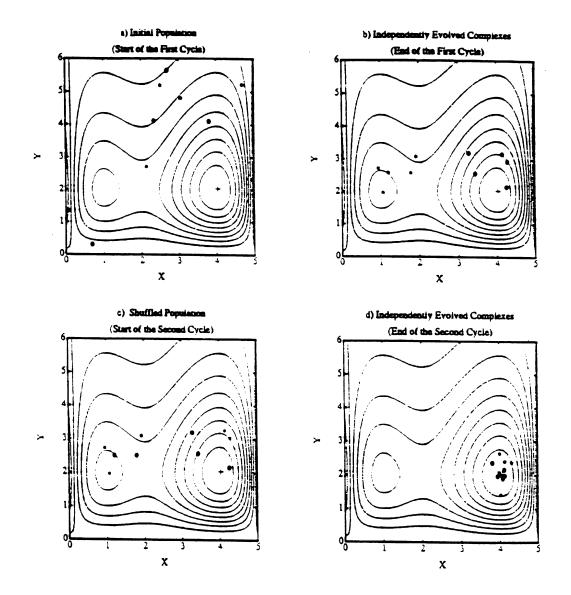


Figure 1. Illustration of the Shuffled Complex Evolution (SCE-UA) Method

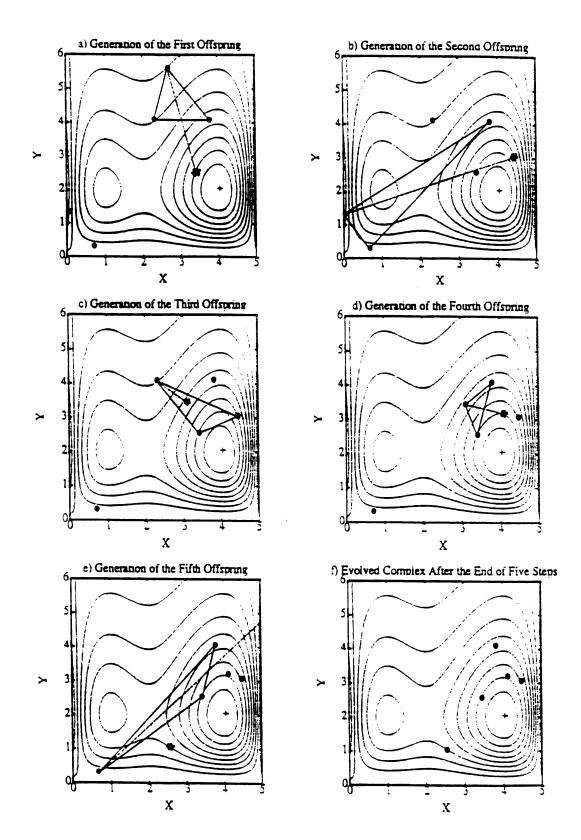


Figure 2. Illustration of the Evolution Steps Taken by Each Complex