VIII.3.3-LAG/K   LAG AND K ROUTING OPERATION

Identifier: LAG/K

Operation Number: 7

Parameter Array: The FORTRAN identifier used for the parameter array
for this Operation is P. The contents of the P array are:

Position	Contents
1	Operation version number
2	Identifier for inflow time series
4	Data type code for inflow time series
5	Data time interval for inflow time series (HR)
6	Identifier for outflow time series
8	Data type code for outflow time series
9	Data time interval for outflow time series (HR)

10	Indicator for units in which data were input:
	  0 = English
	  1 = Metric

11	Transmission loss recession coefficient for Fort Worth transmission loss computations - valid range is > 0 and < 1:
	  0 = do not do Fort Worth transmission loss
	     computations

12	For Fort Worth transmission loss computations - flow below which there is no transmission loss (CMS)
13-15	Unused
16	Number of values in the P array (excluding the space needed for Atlanta K method)

17	Indicator for default carryover:
	  0 = default carryover
	  1 = read some or all of carryover

18	Location in P array of beginning of K information

19	Number of pairs (Lag,Q) in variable Lag table:
	  0 = constant Lag
	  >0 = number of pairs

```
     Position        Contents

        20          If P(19) = 0, constant lag
                    If P(19) >0 0, P(19) pairs (Lag,Q) in variable lag
                    table

      P(18)         Number of pairs (K,Q) in variable K table:
                       0  = Constant K
                       >0 = number of pairs

     P(18)+1        If P (P(18)) = 0, constant K
                    If P (P(18)) > 0, P(P(18)) pairs (K,Q) in variable
                    K table
```

   The following values are stored in the P array only for the
   Atlanta method of computing K (IATL=1):

```
    P(16)+1         The number of pairs of values in the O versus
                    2*S/DT+O table

    P(16)+2         P(P(16)+1) pairs (O,2*S/DT+O) in the table

    P(16)+2+        The number of pairs of values in the O versus
   2*P(P(16)+1)     2*S/(DT/4)+O table

    P(16)+2+        P(P(16)+2+2*P(P(16)+1)) pairs (O,2*S/(DT/4)+O)
   2*P(P(16)+1)+1   in the table
```

The minimum size of the P array is 22.

Carryover Array: The FORTRAN identifier for the carryover array is
C. The contents of the C array are:

The first location in the C array contains a single value specifying
the length of the C array.

The next three positions of the C array contain:
     1. the current lagged inflow
     2. the current outflow
     3. the current storage

If K is not enabled, the inflow and storage will be set to zero and
will not be used in any calculations.

If K is enabled and period average inflows are used the inflow will
be set to zero and will not be used in any calculations.

The Lag portion of the carryover array begins in position 5 of the C
array.

The Lag portion of the C array contains:
     1. a single value which specifies how many pairs (Q,T) of lag
         carryover values will follow
     2. the lag carryover values (number = (longest lag/Δt + 2) * 2)

The minimum size of the C array is 4. The maximum size of the C array depends on the number of Lag carryover values to be stored.

Subroutine Names and Functions:

Subroutine	Function
PIN7	Input cards and store values in P and C arrays
PINA7	Compute the 2*S/DT+O versus O and 2*S/(DT/4)+O versus O tables needed by the Atlanta K Procedure and stores these tables in the P array
FOP7	Check if the Lag and/or K options are on
EX7	Execute the Operation
FEXA7	Control the LAG/K looping - called by EX7 after working space is partitioned among several time series
FLAG7	Control the Lag Operation
FK7	Control the attenuation (K) Operation for the MCP2 method
FCMPK7	Perform the attenuation Operation for the MCP2 method
FQDT7	Called if the MCP variable K is enabled and K found from interpolation of K,Q table is LT routing interval/4.0
FSERC7	Linearly interpolate in the second column of a two dimensional matrix and returns the value from the first column which corresponds to the location in the second column which matches a supplied value
FINTP7	Linearly interpolate in the second column of a two dimensional matrix and return the value from the first column which corresponds to the location in the second column which matches a supplied value - used in place of function FSERC7 for the multiple intercept option only
FKA7	Perform the attenuation (K) computations for the Atlanta method
FTWL7	Perform the Fort Worth transmission loss computations
PRP7	Print the parameters for the LAG/K Operation
FPRLQ7	Print a Lag versus Q table

Subroutine	Function
FPRKQ7	Print a K versus Q table
PRC7	Print the LAG/K carryover
FPRQT7	Print a Q versus time table
PROS7	Print the 2*S/DT+O versus O table
FPRPC7	Print the P and C arrays
PUC7	Output card images in a format that routine PIN7 can read to reproduce the existing LAG/K Operation
COX7	Transfer carryover values from one set of parameters to another
FILLV7	Fill a portion of a real array with a specified value
FTRNR7	Copy a portion of one real array into another real array
FCBLOC07	Set values in common block FATLGK

Subroutine PIN7, PRP7,PRC7, COX7 and PUC7 have the standard argument lists for these subroutines as given in Section VIII.4.3.

SUBROUTINE EX7 (P,C,QA,QB)

<u>Function</u>: This routine performs the Lag and/or K functions and
returns a time series of outflow in the time step equal to the time
step of the inflow time series.

This routine can compute variable K using the method developed by the
Atlanta RFC if the variable IATL in common block FATLGK is set to
one. This method solves the storage equation by using the
relationship between 2*S/DT+O and O. This relationship is
constructed from the variable K curve using the equation
delta_S=K*delta_0. The variables C1 and C2 in common block FATLGK
are used to specify the delta_O increments used in computing the
2*S/DT+O versus O table.

| | Input/ | | | |
Argument	Output	Type	Dimension	Description
P	Input	R*4	Variable	The P array
C	Both	R*4	Variable	The C array
QA	Input	R*4	Variable	The inflow time series
QB	Output	R*4	Variable	Work array - the first portion of this space will contain the routed inflow time series at end of execution

```
SUBROUTINE TAB7 (TO,LEFT,IUSET,NXT,LPO,PO,LCO,CO,TS,MTS,NWORK,NDD,
                 LWORK,IDT)
```

Function: This is the Operations Table entry subroutine for the
LAG/K Operation.

Argument List: The arguments for this subroutine are similar to the
arguments for the Operation table entry subroutines for other
Operations. A description of the arguments is contained in section
VIII.4.2-TAB.

Operation Table Array: The contents of the TO array are:

Position	Contents
1	The number of this Operation
2	Location in the T array of the next Operation to be executed
3	Location of the parameter array for this Operation in the P array
4	Location of the carryover array for this Operation in the P array
5	Location of inflow data in the D array
6	Location of outflow data in the D array: 0 = if routing at a point
7	Location of working space for routed inflows and intermediate calculations in the D array
8	Location of working space in the D array for changing the time interval: 0 = not needed

Alternative Attenuation (K) Computations

An alternative attenuation (K) computation is included in the LAG/K
Operation. These computations can be used as an alternative to the
default (Atlanta RFC) method but cannot be set by an input run time
option. It will be used if the variable IATL in common block FATLGK
is set to one. The input is the same for both methods except that
the carryover value of storage at the beginning of the routing
interval (read on card 6) is replaced by the inflow at the previous
time interval.

Comparisons between the two methods have shown that the Atlanta RFC
method is less sensitive to the length of the routing interval and
therefore has been selected as the K algorithm used in LAG/K
Operation.

Storage Routing Equation

Storage routing with K attenuation is based on the continuity
equation expressed as

$$I - O = \frac{dS}{dt} \tag{1}$$

or as

$$\bar{I} - \bar{O} = \frac{\Delta S}{t_r} \tag{2}$$

where I represents inflow rate, O is outflow rate, S is storage and t
is time with t_r representing the routing interval. Using subscripts 1
and 2 to represent the beginning and end of routing interval t_r, the
equation becomes

$$\frac{I_1 + I_2}{2} t_r - \frac{O_1 + O_2}{2} t_r = S_2 - S_1 \tag{3}$$

In a typical routing situation, both inflows and the initial outflow
and storage values are known. The unknowns in the equation are O_2 and
S_2. Since K is a ratio of storage to discharge, it can be entered
into the equation leaving only O_2 as an unknown quantity because

$$\Delta S = S_2 - S_1 \tag{4}$$

$$\Delta S = KO_2 - KO_1 \tag{5}$$

$$KO_2 - KO_1 = .5t_r(I_1 + I_2 - O_1 - O_2) \tag{6}$$

Solving for O_2, the routing equation used in Subroutine FK7 becomes

$$O_2 = .5t_r \frac{I_1 + I_2}{K + .5t_r} + O_1 \frac{K - .5t_r}{K + .5t_r} \tag{7}$$

O_2 is therefore a function of I_1, I_2, O_1 and K.

Routing Intervals

Accuracy in the routing equation is affected by the length of the routing interval t_r, with shorter routing intervals providing greater accuracy. The attenuation algorithm automatically creates a routing interval which is one-half of the ordinate spacing ($\triangle t$) or t=$\triangle t$/2. Routing occurs with time step=t_r, but values are placed in the outflow time series, QB, only at $\triangle t$ intervals. Accuracy is improved by making the routing interval one-half of the ordinate spacing regardless of whether K is constant or variable.

Estimation of Average Outflow

The routing equation assumes that K remains unchanged during the routing interval. This assumption is true when K is constant, but can be untrue when K is variable. Some errors are therefore unavoidable. Routing errors are more likely and greatest when inflows rise and fall sharply and when the variable K relationship contains wide variations. The attenuation procedure in subroutine FKT attempts to minimize the potential for error inherent in the equation in two ways. First, the routing interval is one-half of the ordinate spacing as described in the previous paragraph. This produces a shorter routing interval which has to be represented by one value of K. Presumably, outflow will be less variable during a shorter interval. Second, K is based on average outflow (O_μ) during the routing interval instead of being based on outflow at the beginning of the interval (O_1). K is thus more representative of the range of outflow which occurs during the interval. However, since O_2 is an unknown quantity, it is necessary to make an initial estimate of average outflow (O_μ) from known quantities at the beginning of the routing interval, including O_1 and the previous outflow value. This is accomplished via linear extrapolation in the following manner:

$$\hat{O}_\mu = \frac{3O_n - O_{n-1}}{2} \tag{8}$$

where O_n represents a position in array QB equivalent to O_1. Following this estimation, a value of K based on O_μ is determined from the K versus outflow relationship. After routing and finding a value for O_2, O_μ is compared with O_μ based on O_1 and O_2. If a significant difference exists, the routing process is repeated with K based on the new computed average outflow instead of O_μ. This process is repeated as necessary until the error between the original and re-computed average outflows has been reduced below the tolerance. In most cases, the first estimation of average outflow is sufficiently accurate to preclude iteration of this process. In extreme cases, where K changes dramatically with minor changes in outflow, an accurate estimate can still be found within two or three iterations. Routed values are not entered into the outflow array until an accurate estimate of average outflow has been found and until K has been satisfactorily determined.

Limitations of the Attenuation Procedure with K

The routing interval t_r, where $t=\Delta t/2$, prohibits the use of very small values of K in the attenuation operation. When a value of K less than one-half of the routing interval is applied to the routing equation, it can produce incorrect results. This situation is handled differently for constant K and variable K. Each will be addressed separately.

With constant K no attenuation is performed when K is less than one-fourth of the ordinate spacing Δt. For example, assume that the ordinate spacing for a flow-point is 6 hours. If one wanted to attenuate the flow with a constant K, then K would have to be greater than or equal to 1.5 hours. If one attempted to use a constant K less than 1.5 hours in this situation, the routing equation would be bypassed, no attenuation would occur and outflow would equal inflow. The only way to apply a constant K of less than 1.5 hours would be to change the ordinate spacing. A reduction in Δt from 6 hours to 3 hours would permit the use of constant K values greater than or equal to 45 minutes.

Most variable K relationships have K values which are usually greater than one-half of the routing interval t_r. A substantial amount of attenuation is normally applied to all flows in these cases. However, the problem of K being less than $.5t_r$ can occur since relationships where K varies from 0.0 to values greater than 0.0 are permitted. The transition of K from 0.0 to values greater than 0.0 means that certain flows will require a K value less than $.5t_r$. The K routing procedure tries to make this transition smooth and accurate. When a value of K between 0.0 and $.5t_r$ is interpolated from the variable K relationship, the following actions are taken:

1. The initial routing interval is quartered for the one time interval requiring attenuation where $0.0< K <.5t_r$. This creates a short interval routing sequence with four short intervals taking the place of the original interval t_r.

2. Additional inflow values are found through linear interpolation. Additional outflow values needed to estimate mean outflow are obtained through linear extrapolation.

3. Sequential routing takes place within the shorter routing intervals. Four routing operations are required to determine the sought-after outflow value.

Quartering of the routing interval in this manner permits values of K to be used which are greater than or equal to one-eighth of the routing interval t_r. For example, when $\Delta t=6$ hours and $t_r=3$ hours, attenuation by as small an amount as 22.5 minutes is possible. Values of K during the short interval routing sequence which are less than one-eighth of the original t_r are not applied and outflow is set to equal inflow. Thus, attenuation with K equal to 15 minutes would not be possible in this example.

Reference

Linsley, R.K., Kohler, M.A. and Paulhus, J.L.H., 1975: Hydrology for
Engineers, 2nd ed. McGraw Hill Book Company, New York.