FLDWAV APPLICATION:
TRANSITIONING FROM CALIBRATION TO OPERATIONAL MODE

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INTRODUCTION

The National Weather Service provides river forecasts of our Nation's streams. River forecasts are often complicated by man-made structures (e.g., dams, bridges, levees, etc.); unsteady flows subjected to backwater effects caused by reservoirs, tides, or inflows from large tributaries; and mild channel bottom slopes where flow inertial effects are important. Within the National Weather Service River Forecast System (NWSRFS), the runoff generated by rainfall-runoff models aggregates into fairly large, well-defined channels, and then is transmitted downstream by routing techniques. The hydrologic or storage routing methods, e.g. the Lag and K technique (Linsley, et al., 1958) may be adequate for many situations; however, hydraulic routing methods are necessary for the previously described conditions. The Hydrology Laboratory (HL) of the NWS Office of Hydrologic Development (OHD) has developed dynamic wave routing models suitable for efficient operational use in a wide variety of applications involving the prediction of unsteady flows in rivers, reservoirs, and estuaries (Fread, 1992). The NWS Flood Wave routing model (FLDWAV) (Fread and Lewis, 1988), the latest unsteady flow model developed at HL, provides real-time forecasts of discharges, water-surface elevations, and velocities at specified locations along a river and its dynamically-modeled tributaries.

Operational models cover a 24 hrs/day - 365-days/year period. Critical components to timely and accurate forecasts are model stability and universal applicability. Unsteady flow models are often unstable when applied to complex river systems; however FLDWAV has several automatic features, which increase model stability. FLDWAV has been implemented on major rivers in the United States including the Mississippi River, Columbia River, Red River of the North, Ohio River and Susquehanna River. Prior to implementation, calibration of model parameters for various flow conditions ensures accurate results from FLDWAV. By accounting for operational concerns during the calibration process, FLDWAV becomes a powerful tool for generating accurate river forecasts. This paper gives an overview of the calibration and implementation processes as applied to a portion of the Susquehanna River System in Pennsylvania.

DESCRIPTION OF FLDWAV

The NWS FLDWAV model is an unsteady-flow, dynamic, hydraulic routing model which determines the water-surface elevation and discharge at specified locations along the length of a waterway (river, reservoir, etc) subjected to an unsteady flow event such as a flood wave or dam-break wave. The model is based on an implicit (four-point, nonlinear) finite-difference solution of the complete one dimensional Saint-Venant unsteady flow equations coupled with an assortment of internal boundary conditions representing unsteady flows controlled by a wide spectrum of hydraulic structures. The flow may occur in a single waterway or a system of inter-connected waterways, including those having dendritic structures (nth-order tributaries) in which sinuosity effects are considered. Additional capabilities of FLDWAV include: 1) the ability to dynamically model dam failures as well as flows which are affected by bridge constrictions; 2) the ability to simulate flows which overtop and crevasse levees located along either or both sides of a main stem and/or its principal tributaries; and 3) the ability to handle flows in the subcritical and/or supercritical flow regime.

The expanded Saint-Venant equations of conservation of mass and momentum consist of the following (Fread and Lewis, 1993):

$$\partial Q/\partial x \ + \ \partial s_c(A + A_o)/\partial t - q = 0 \qquad (1)$$

$$\partial(s_m Q)/\partial t + \partial(\beta Q^2/A)/\partial x + gA(\partial h/\partial x + S_f + S_e) + L + W_f B = 0 \qquad (2)$$

in which Q is discharge (flow), A is wetted active cross-sectional area, A_o is wetted inactive off-channel (dead) storage area associated with topographical embayments or tributaries, B is the channel flow width, s_c and s_m are depth-dependent sinuosity coefficients for mass and momentum, respectively, that account for meander, $ is the momentum coefficient for nonuniform velocity, q is lateral flow (inflow is positive, outflow is negative), t is time, x is distance measured along the mean flow-path of the floodplain, g is the gravitational acceleration constant, h is the water-surface elevation, L is the momentum effect of lateral flows ($L=-qv_x$ for lateral inflow where v_x is the lateral inflow velocity in the x-direction, L=-qQ/(2A) for seepage lateral outflows, L= -qQ/A for bulk lateral outflows such as flows over levees), S_f is the boundary friction slope ($S_f=(Qn/(1.49AR^{2/3}))^2$ where n is the Manning roughness coefficient and R is the hydraulic radius), S_e is the slope due to local expansion-contraction (large eddy loss), and W_f is the wind term.

The information necessary to execute FLDWAV includes: 1) an upstream stage or discharge hydrograph; 2) a downstream boundary condition (stage hydrograph or a rating curve); 3) cross section geometry (top width vs. elevation table); 4) information about hydraulic structures (dams, bridges, levees); 5) hydraulic roughness coefficients which may vary with elevation or discharge and with location along the waterway; and 6) the initial h and Q at each cross section location. Given this information, FLDWAV simultaneously solves for the h and Q at each cross section location along the routing reach for each time interval during the specified simulation period using the unsteady flow equations.

CALIBRATION PROCESS

Data Requirements: When calibrating a river system, FLDWAV must be set up with specific data to ensure that roughness coefficients are adequately calibrated such that the forecast water surface elevations (WSEL) may be as accurate as those calibrated.

River System: A river system consisting of one or more rivers is defined by its boundary conditions, gaging stations, lateral/tributary flows, and cross section topography. Rivers influenced by backwater conditions, with very flat bottom slopes (less than about 2-3 ft/mi), having rapidly varying temporal changes in the flow, with forecast points, and/or where flood inundation mapping is desired are dynamic rivers through which flow is routed using the unsteady flow equations. All other rivers in the system are treated as lateral (local) inflows, i.e., the flow is added to the river system without being routed. Local flows include both gaged and ungaged flow. Since the objective is to forecast accurate river levels, discharge hydrographs generated within NWSRFS are preferred to flows from other sources (e.g., U. S. Geological Survey (USGS)) because any errors in the computed discharges will be accounted for in the calibration process.

Flood Simulations: To capture the widest range of flow, two floods should be used to calibrate the model: the flood of record to capture the maximum flood condition, and a minimum flood to capture the low-flow condition. Another flood should be simulated to verify or determine how well the model will perform on an independent data set. This is an indicator of how well the model will perform in forecast mode. To account for the beyond-flood-of-record condition, the maximum flood is doubled. The cross section and Manning n tables are adjusted to accommodate this super flood condition. To model the drought condition, all local flows are set to zero and inflows are set to minimum values to prevent model instability.

Upstream Boundary: The upstream boundary condition is normally a discharge hydrograph. The upstream boundary location must be identified for each dynamic river. This location should be far enough upstream where the influence of downstream backwater conditions is not felt.

Downstream Boundary: The downstream boundary condition on the main river must be reproducible in the forecast mode. Typically it is either an empirical single-valued rating curve or a generated loop-rating curve. Under some backwater conditions (e.g., backwater from a downstream river), the rating curve is not adequate to represent the stage-discharge relationship; therefore, the downstream boundary is moved far enough away until it has no influence on the last point of interest. The final reach may be either a fictitious reach manipulated to produce the best results at the last point of interest, or the reach to the next downstream gage. The downstream boundary condition for tributaries is a WSEL hydrograph generated within FLDWAV.

Forecast Points: Gaging stations are locations where observed stage data are available. Forecast points are locations where the NWS issues a river forecast. All forecast points may not be gages. Since FLDWAV generates stages and discharges at all computational points in the river system, this information is available at the forecast points.

Manning Roughness Coefficients: Manning's n is used to describe the resistance to flow caused by channel roughness resulting from sand/gravel bed-forms, bank vegetation and obstructions, bend effects, and circulation-eddy losses. The reach between adjacent gages is a Manning's n reach. The roughness coefficients are usually input as a function of discharge.

Cross Sections: Cross sections are used to describe the channel/valley. They should be located along the river such that they adequately define the topography (e.g., expansions and contractions, flood storage, etc.). The distance between cross sections should obey the Courant condition for model stability:

$$\Delta x \leq c\, \Delta t \tag{3}$$

where)x is the distance interval between cross sections, c is the wave speed celerity, and)t is the time increment. Additional cross sections may also be generated by the automatic interpolation in

FLDWAV to maintain model stability. For major rivers, the U. S. Army Corps of Engineers is a good source of surveyed cross section data. If survey sections are unavailable, the floodplain portion of the section can be obtained from USGS topographic maps or from DEM data. In the absence of surveyed data, the channel portion of the cross section may have to be estimated.

Flow Regime: The flow regime for most rivers is subcritical flow. However, because of the very wide range of flow conditions encountered during real-time forecasting, there may be times when the flow regime may change to critical or supercritical flow. The local partial inertial (LPI) technique (Fread et al, 1996) in FLDWAV adds a factor to the inertial term in the momentum equation, which allows the river to change flow regimes anywhere in the channel reach when necessary during the simulation period. In river systems where the minimum depth of water in the channel is less than three feet, the mixed flow option using the LPI technique should be used.

FLDWAV Calibration: In order to produce an acceptable forecast using FLDWAV, the model must first be calibrated by adjusting the roughness coefficients until the computed and observed stage hydrographs match at each gage. After selecting an appropriate historical flood and acquiring the data, the FLDWAV model is used to calibrate the Manning n values as follows: 1) estimate the Manning n values throughout the routing reach; 2) run FLDWAV and compare the computed WSEL with the observed WSEL at each gage; 3) adjust the Manning n values to accommodate the errors in WSEL; 4) repeat steps (2) and (3) until the error root-mean-squared error (RMSE) has been minimized. The final n values are sufficient for the range of flows used in the calibration; however, the Manning n values for flows exceeding the observed must be estimated.

RUNNING FLDWAV OPERATIONALLY

Initial Conditions: The correct initial conditions are critical to effective real-time forecasting using FLDWAV. In FLDWAV, the initial conditions are: the initial water surface elevations and discharges at each computational point; initial flow at each lateral flow point; and initial pool elevations and gate control switches at each dam location. A forecaster usually starts a simulation three to five days prior to the current date. During simulation, this information is generated within FLDWAV and stored for future runs. If FLDWAV becomes unstable, the initial conditions are corrupted and future forecasts are in jeopardy. Therefore, great care is taken during calibration to ensure model stability.

Real-Time Forecasting Numerical Difficulties: Model stability may be a problem when running FLDWAV in real-time especially during low flow periods. To prevent the model from terminating abnormally, a minimum-flow filter is specified on each river.

There are times when the model may abnormally terminate due to non-convergence (i.e. an adequate solution to the Saint-Venant Equations has not been obtained within the user-specified number of iterations, automatic fix-ups (Fread, 1988) were unsuccessful, and the results were extrapolated more than six times). These non-convergences usually occur as a result of abrupt changes in the flows and/or stages in the river system. When this occurs, the FLDWAV simulation stops and the remaining hydrographs are filled with constant values (stages, discharges, velocities) equal to the last computed values. In many cases non-convergence occurs because the model has not been adequately calibrated; the time step or distance step is not set properly; there are errors in input hydrographs (e.g., erroneous pool elevations; missing data treated as actual); or the flow regime is not adequately defined. The forecaster must then adjust model parameters to fix the problem of non-convergence.

APPLICATION TO THE SUSQUEHANNA RIVER SYSTEM

The Susquehanna River System (Figure 1) is a 530-mile long river system with the Susquehanna River (320 miles from Conklin, NY to Marietta, PA) as the main stem. There are three dynamic tributaries to the Susquehanna River: the Chemung River (177 miles from Chemung, NY to its mouth), the West Branch of the Susquehanna River (39 miles from Williamsport, PA to its mouth), and the Juniata River (48 miles from Lewistown, PA to its mouth); and one dynamic tributary to the Juniata River: Kishacoquillas Creek (6 miles from Reedsville, PA to its mouth). The river system has been divided into three sub-systems with break points at Wilkes-Barre, PA (river mile 186) and Harrisburg, PA (river mile 86). Hydraulic features within the river system include levee overtopping in the vicinity of Harrisburg, PA; backwater due to inflows of large tributaries (e.g., there are times when the West Branch backs up into the Susquehanna and other times when the Susquehanna backs up into the West Branch; the forecast point at Danville, PA is influenced by both conditions); an inflatable dam in the vicinity of Sunbury, PA for recreational purposes; and water depths less than one foot below Harrisburg, PA during the summer months. The town of Lewistown, PA has been selected as a test site for the NWS flood forecast mapping application. For this purpose a two-river system (Figure 2) is modeled separately using FLDWAV. The implementation of the Juniata River system is discussed in this section.

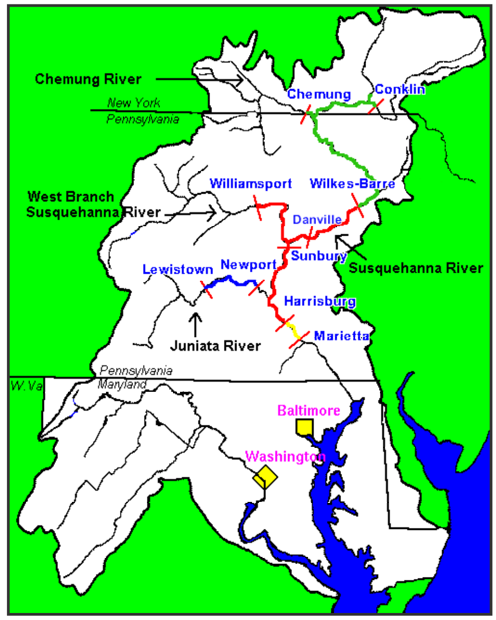

Figure 1 Susquehanna River System

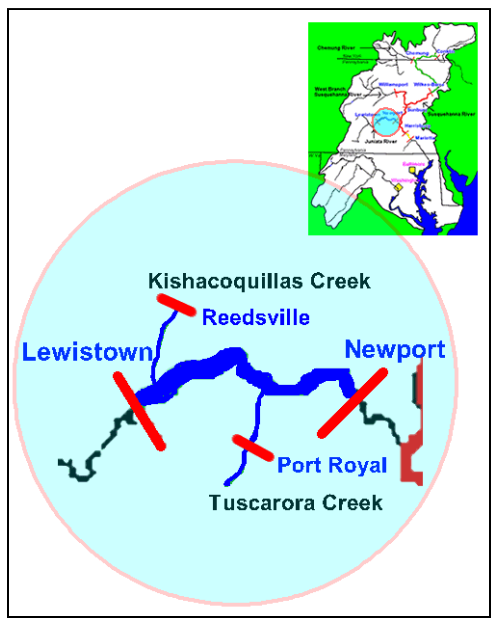

Figure 2 Juniata River System

Lewistown River System: The town of Lewistown, PA covers about two miles of the Juniata River with the Kishacoquillas Creek running through the town. During high flows, the Kishacoquillas Creek is subjected to backwater from the Juniata River. The upstream boundary location is set at the extent of the town, approximately 1.5 miles upstream of the Lewistown gage. The next gage below Lewistown is Newport, PA, which is the downstream boundary location. Tuscarora Creek and the ungaged local flow located between Lewistown and Newport is treated as a lateral flow. The mixed-flow regime is set to accommodate both the flood and drought conditions. A schematic of the river system is shown in Figure 3.

Flood Simulations: Although Hurricane Agnes in 1972 is the maximum known flood (~160,000 cfs) in the area; the gage at Lewistown did not come into existence until 1989. The maximum flood of record for this gage is the 1996 flood (74,400 cfs). The 1997 flood contains the minimum flood of record (580 cfs). The 1996 and 1997 floods are calibrated and the 1999 flood is used for verification. The 1972 flood is also simulated although no elevation data is available. To account for the beyond-flood-of-record condition, the1972 flood is doubled. To account for the drought condition, the local flow at Tuscarora Creek is set to zero, and the inflows at Lewistown and Reedsville are set to constant values of 50 cfs and 1 cfs, respectively. Figure 4 shows the flow range (0-75,000 cfs) for which the Manning n values have been calibrated and estimated (beyond 75,000 cfs). A computational time step of 0.25 hr is selected based on the time to peak of the sharpest rising hydrograph.

Upstream Boundary: The inflow hydrograph at Lewistown is moved to the upstream location of the Juniata River, and the inflow at Reedsville is the upstream boundary for Kishacoquillas Creek.

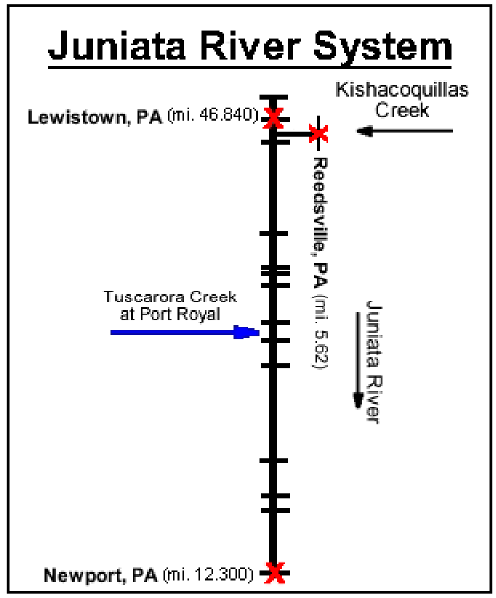

Figure 3 Schematic of Juniata River System

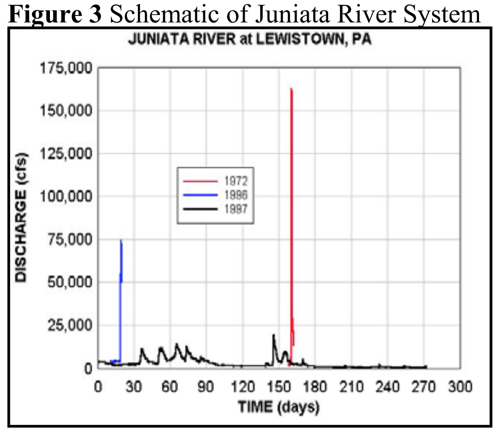

Figure 4 Discharge Hydrographs at Lewistown, PA for 1972, 1996, 1997 Floods

Downstream Boundary: To determine if the downstream boundary is affected by backwater from the Susquehanna River, various boundary conditions are analyzed. A typical downstream boundary condition in FLDWAV is the generated rating curve, which represents the dynamic effects due to unsteady flow. The observed rating curves at Newport for the 1996, 1997 and 1972 floods (Figure 5) show that the stage-discharge relationship is not linear, which indicates that dynamic effects are present. The rating curves generated by FLDWAV indicate that the dynamic effects due to unsteady flow are minimal since the stage-discharge relationship is linear. The peak stage is underestimated by 15 ft (Figure 6). The 15-foot error occurs because the rating curve does not account for the

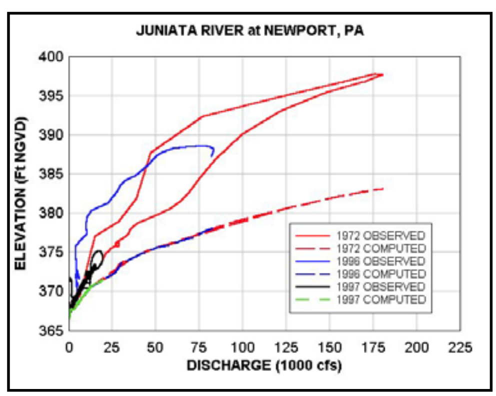

Figure 5 Observed vs. Computed Rating Curve, Newport, PA

Susquehanna River backwater effect. When the Juniata River is modeled as a tributary to the Susquehanna River, the peak stage computed by FLDWAV matches the observed stage with a difference of 0.41 ft. The computed discharge hydrograph at Newport is the same regardless of the downstream boundary condition (maximum error of 1.5%). Since the Lewistown gage is the point of interest, rating curves are plotted to see if the backwater effects are felt at the gage. Figure 5 shows that the rating curves for various floods are essentially the same when the computed and observed rating curves are compared. Therefore, the backwater effects do not reach Lewistown, and the generated rating curve is an adequate downstream boundary condition.

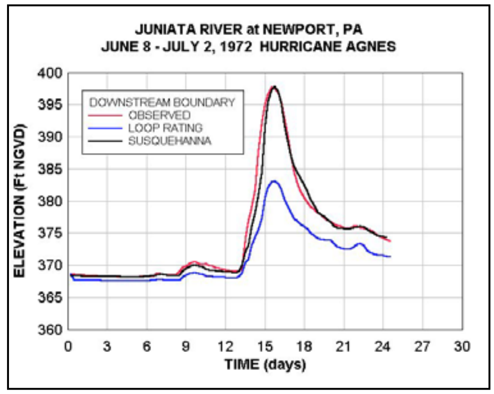

Figure 6 Downstream Boundary Comparisons

Cross Sections: In the absence of surveyed cross sections, 7.5 minute USGS topographic maps are used to determine the cross sectional data. The cross sections are initially placed to account for changes in the topography (e.g., transitions between constrictions and expansions) and structures (e.g., bridges and levees). Although Highway 522 (Figure 7) could be considered a levee, water on the town side is caused by the Juniata River backing water up into Kishacoquillas Creek, which then floods the town. Therefore the area beyond the highway is considered to be part of the Kishacoquillas Creek topography. There are also bridges and railroads in the routing reach, but their impact on the water level do not require them to be modeled in a special manner with FLDWAV. Any effects of these structures are accounted for by the roughness coefficient.

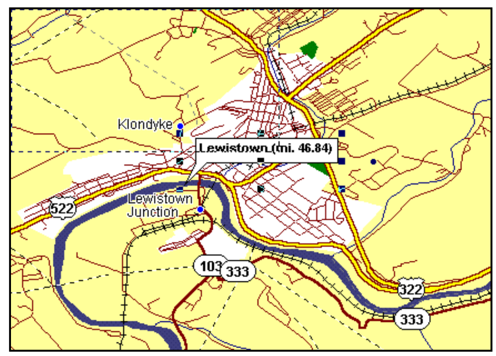

Figure 7 Lewistown, PA (Ref: Tiger Map, US Census Bureau)

Fourteen cross sections on the Juniata River and three cross sections on Kishacoquillas Creek are selected. Using the Courant condition as a guide, a distance interval of 1 mile is used for the Juniata River and 0.2 mi is used for Kishacoquillas Creek to ensure model stability. After FLDWAV adds cross sections by interpolation, the river system has 19 cross sections on the Juniata River and 47 cross sections on Kishacoquillas Creek

Since topographic maps do not represent the channel bottom, the minimum elevation obtained from the topographic map is lowered 5 ft. During calibration, the channel bottom at each of the cross sections in the reach is adjusted in conjunction with the roughness coefficients in an effort to reproduce the observed stages at Lewistown. The cross section at Lewistown (Figure 8) and the FLDWAV representation of channel width vs. elevation (Figure 9) are shown.

During calibration for the minimum and maximum floods, the channel bottom at Lewistown is lowered about 12 ft. Table 1 shows the minimum and maximum discharges and elevations for the various floods modeled using FLDWAV in addition to the observed values. The minimum observed

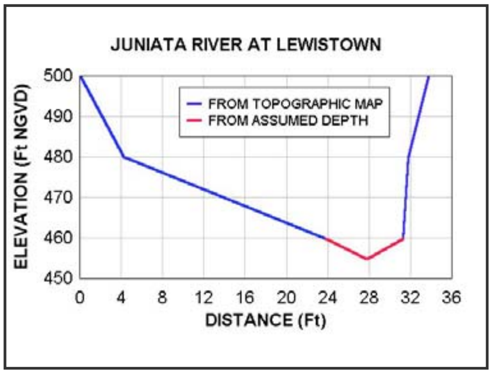

Figure 8 Estimated Cross Section at Lewistown, PA

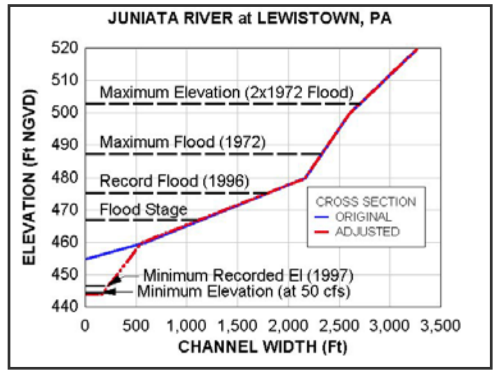

Figure 9 Channel Width vs. Elevation Curve

elevation results in a 2.73 ft channel depth. When the minimum allowable flow (50 cfs) is routed through the channel reach, the corresponding depth at Lewistown is 0.5 ft. The maximum observed depth is 31.57 ft. Hurricane Agnes is the flood of record at other gages in the vicinity of Lewistown, which have a longer recording history (e.g., Newport); therefore, it is necessary to ensure that the cross sections in the routing reach can accommodate the 1972 flood. The 1972 flood yields an additional 12 ft of depth beyond the flood of record. To ensure that the cross sections can contain a super flood, the 1972 flood is doubled. The result is an additional 15.4 ft of depth beyond the 1972 flood. Since the cross section at Lewistown has a maximum elevation of 500 ft and the maximum elevation of the super flood is 502.92 ft, the cross section is extended to 520 ft. When the validation flood (1999) is compared with the other floods, it is very similar in magnitude to the 1997 flood. The minimum elevation is about 0.4 ft lower than the 1997 minimum flood level; however no modifications are necessary to the cross section since the minimum modeled level is about 2 ft lower. The cross sections in the routing reach have been adequately sized to model both a severe drought and a significant super flood without numerical difficulties.

Table 1 Elevations and Discharges at Lewistown, PA

Flood Year	Elevation (FT-NGVD)				Discharge (CFS)	
			Observed			
	Min	Max	Min	Max	Min	Max
1996	449.86	475.08	449.67	475.41	3,820	73,920
1997	446.56	457.64	446.56	457.92	570	19,750
1972	448.37	487.48	--------	--------	1,900	161,160
1999	446.17	458.01	446.34	457.54	420	20.73
Max	449.82	502.92	--------	--------	3,790	322,550
Min	444.43	444.43	--------	--------	50	50

Channel Invert: 443.83 ft Gage Zero: 443.83 ft Flood Elev: 466.95

Roughness Coefficients: The river system has one Manning n reach on the Juniata River between Lewistown and Newport, and one reach on Kishacoquillas Creek between Reedsville and the mouth. A constant Manning n coefficient equal to 0.035 for both reaches is initially used to simulate the 1996 flood. Manning n is a function of discharge. The simulated elevations (using constant and calibrated Manning n values) at Lewistown (Figure 10) are similar to the observed values until the discharges exceed 50,000 cfs where the peak elevation is over computed by 5.57 ft. The Manning n values in the flow range of 0-75,000 cfs are calibrated with the 1996 and 1997 flood data. A constant Manning n value of 0.55 is used in the flow range greater than 75,000 cfs, which is a reasonable value, based on topography. This flow range is not calibrated since no observed elevation data is available. Figure 11 represent the calibrated Manning n vs. discharge curve. Kishacoquillas Creek is controlled by backwater from the Juniata River. The Manning n curve for the Juniata River is also used for Kishacoquillas Creek because flow contributed by Kishacoquillas Creek and controlled by its Manning n values is minimal compared to backwater effects.

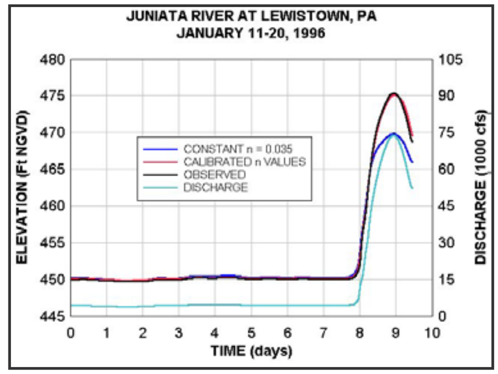

Figure 10 Elevation Hydrographs Comparing Manning n Curves, 1996 Flood

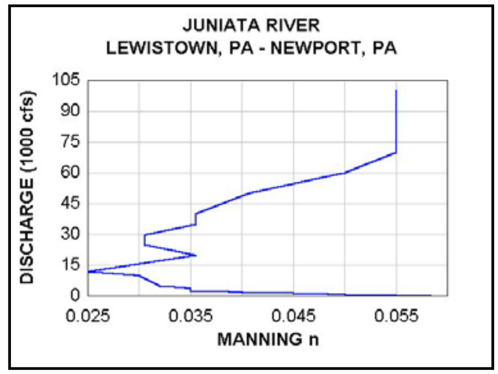

Figure 11 Calibrated Manning n Curve

Calibration Results: The Manning n values are calibrated using FLDWAV for the 1996 and 1997 floods. Figures 10 and 12 compare the computed vs. observed elevations at Lewistown for the 1996 and 1997 floods, respectively. Table 2 shows the errors associated with the floods. The RMSE for the 1996 flood is .27 ft. Although the RMSE show little change for the 1997 flood, the minimum elevation error improves to 0.19 ft.

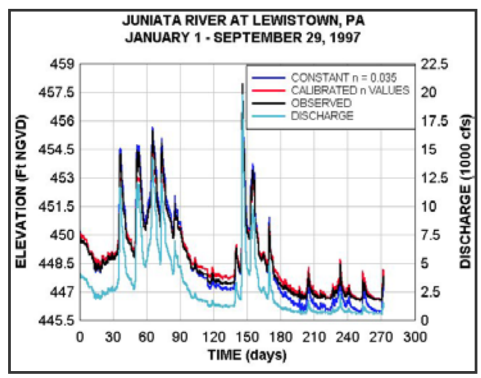

Figure 12 Elevation Hydrographs Comparing Manning n Curves, 1997 Flood

Table 2 Error Analysis for Lewistown, PA Gage

Flood Year	RMSE (ft)		Error in Peak Elevation		Error in Min Elevation	
	Manning n		Manning n		Manning n	
	Const	Calib	Const	Calib	Const	Calib
1996	1.56	0.27	5.57	-0.33	0.19	0.00
1997	0.36	0.31	0.29	-0.27	-0.61	0.19
1999	-----	0.61	-----	0.47	-----	0.17

Verification Results: The 1999 flood (Figure 13) is simulated using the calibrated roughness coefficients to determine the quality of the calibration. Table 2 shows an increase in error, which is expected. Generally, if the RMSE for the calibrated run is less that 0.5 ft and the RMSE for an independent flood is less than one foot, then the FLDWAV model is performing satisfactorily. The results of the current calibration and verification meet this criterion.

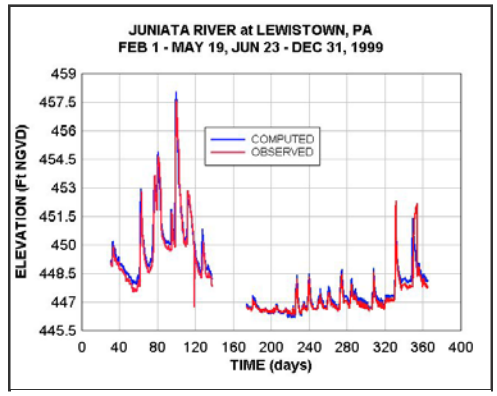

Figure 13 Water Surface Elevations, 1999 Flood

<u>Operational Forecasting</u>: When transitioning from calibration to operational mode, the downstream boundary on the Juniata is changed from a water surface hydrograph to a generated rating curve. The water surface profile on the Juniata River for the 1972 flood (Figure 14) shows the impact of changing the downstream boundary has no impact on the area of interest, the vicinity of Lewistown, PA. The FLDWAV model as applied to the Juniata River system is very stable with no extrapolations and very few non-convergence problems for any of the flood simulations used. This indicates that in an operational mode, the model should run to completion with reasonable results for the river forecast.

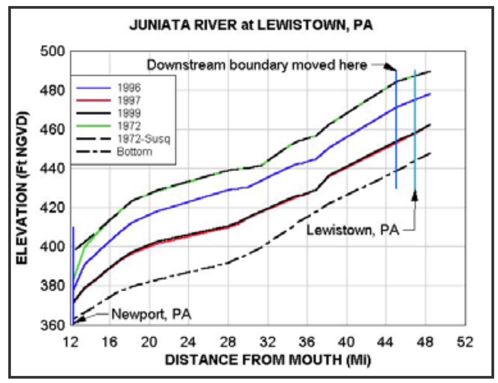

Figure 14 Water Surface Profiles

Initial Conditions: At the time of implementation, the Juniata River is experiencing low flow and the initial conditions are assumed to be steady-state backwater condition with normal depth as the initial elevation at the downstream extremity. For subsequent runs, the initial conditions are stored (water surface elevations and discharges) at each cross section location in the river system for as many as nine dates in the past and the current date.

Flood Forecast Mapping: The NWS FLDVIEW application (Cajina et. al., 2002) has been developed to display the extent of flooding based on the forecast water surface profile. The input requirements for FLDVIEW include the water surface profile, the channel width corresponding to the known water surface elevation, the channel bottom, cross section locations, and the latitude/longitude of the end points of the flood map. The FLDWAV model exports this information to files, which FLDVIEW reads in. The 1984 flood is simulated using FLDWAV and the flood forecast map is generated using FLDVIEW. The extent of the flood is shown if Figure 15 along with four high water marks. The high water marks indicate that FLDVIEW adequately represents the extent of flooding.

Figure 15 Flood Forecast Map, Lewistown, PA; 1984 Flood

SUMMARY

The NWS FLDWAV model is a powerful tool when used to produce river forecasts on complex river systems. When properly calibrated it can forecast extreme events (droughts and major floods) as well as intermediate flood events with a high level of accuracy. FLDWAV also has built-in capabilities to maintain model stability, which is critical during operational mode. The accuracy of the FLDWAV results combined with the mapping ability of the new NWS FLDVIEW application enhance the river forecast by providing an accurate flood forecast map, which allows the user to see the extent of flooding.

The FLDWAV model has been applied to the Juniata River system in order to generate flood forecast maps in the vicinity of the town of Lewistown, PA. It has been calibrated using the 1996 and 1997 flood data; and the results validated using the 1999 flood data. The RMSE for 1999 flood is 0.61 ft, which indicates that accurate results will be obtained during forecast mode. The 1984 flood has also been simulated using FLDWAV and the peak water surface profile has been mapped using the NWS flood forecast mapping application (FLDVIEW). The extent of flooding mapped by FLDVIEW compares well with the high water marks for the flood.

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