
NATIONAL WEATHER SERVICE
OFFICE of HYDROLOGIC DEVELOPMENT

ALGORITHM DESCRIPTION DOCUMENT

Sacramento Model Enhancement
To
Handle Implications of Frozen Ground on Watershed Runoff

Version 3.1
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Revision History

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1. INTRODUCTION AND SCOPE

1.1 Identification

"Frozen Ground Sacramento Soil Moisture Accounting (SAC) Model Enhancement Project"

Project ID: OHD-05-002

1.2 Document Overview

Section 1 of this Algorithm Description Document (ADD) is an introductory section.

1.3 General Description of Operational Capability

The Hydrology Laboratory (HL) has begun the development of a new physically-based frozen ground parameterization capability. The first part is the conceptualization of the heat transfer processes. This part is completed and some tests have been performed. Testing using specially developed prototype algorithms was also conducted. The results proved to be supportable and scientifically sound.

The objective of this project is to leverage the work already completed to produce a new Sacramento Soil Moisture Accounting (SAC-SMA) capability. The OHD/HL Hydrologic Science and Modeling Branch (HSMB) has determined that the Sacramento Model can be made more accurate and useful if it uses a physically-based algorithm to compute the effects of frozen ground. A prototype was developed by the HSMB and used to prove the scientific theory. The code has been tested in a LINUX NWS River Forecast System (NWSRFS) Calibration System environment and meets scientific needs. Testing has proven the algorithm and physics to be scientifically correct. The results have been reviewed by National Weather Service (NWS) experts. In addition, the results have been reviewed and approved by other experts via peer-reviewed publications and conference presentations. Test data and results are provided for development. The development effort should provide the same results.

The prototype code, test data, and test results will be provided to the Hydrologic Software Engineering Branch (HSEB). The resulting frozen ground improvement to the SAC-SMA is required to reproduce the submitted test results when testing is conducted using the same test input data to ensure that the scientific integrity is maintained.

Section 1.3 was copied from the Concept of Operations Document (CONOPS).

1.3 Source & References

The physically-based frozen ground algorithm was developed by Victor Koren of the Hydrology Laboratory of the National Weather Service Office of Hydrologic Development. It was designed to replace the conceptual frozen ground component developed by Eric Anderson and Pat Neuman (Anderson and Neuman, 1984).

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- Kulik, V. Ya., 1969. Effects of ice content on the soil conductivity, Meteorology and Hydrology, No 9, 66-71.

- Mitchell, K., M. Ek, D. Lohmann, **V. Koren**, **J. Schaake, Q. Duan**, P. Grunmann, G. Gayno, Y. Lin, E. Rogers, D. Tarpley, C. Peters-Lidard, 2002. Reducing near-surface cool/moist biases over snowpack and early spring wet soils in NCEP Eta model forecasts via land surface model upgrades, 16th Conference on Hydrology, January, 2002, Orlando, FL, AMS, paper J1.1, pp. J1-J6.

2. FUNCTIONAL DESCRIPTION

NWS modeling of frozen ground effects on the rainfall-runoff process began with the development of a conceptual modification to the Sacramento Soil Moisture Accounting Model (SAC-SMA) (Anderson and Neuman, 1984). This approach used the concept of a frost index which modified the runoff generated by the SAC-SMA. As a conceptual model, this approach requires the calibration of 7 parameters.

A successful collaboration between the Office of Hydrologic Development and National center for Environmental Prediction (NCEP) resulted in the development of a heat-transfer component of the NCEP land surface model. As reported by Koren et al. (1999) and Mitchell et al. (2002), this parameterization proved to be a valid component that reduced biases in certain computed values.

Capitalizing on this successful collaboration, the HL began to develop an advanced approach to modifying the SAC-SMA for the effects of frozen ground. This work led to a physically-based heat transfer component for the SAC-SMA that requires minimal calibration. Two basic requirements were identified for this component. The component must:

- Be simple enough to run with sparse and noisy data; and
- Match the complexity of the SAC-SMA.

A conceptual representation of soil moisture fluxes combined with a physically-based heat transfer model provides reasonable simulations of water and heat exchanges over a soil profile. Physically-based modification of the hydraulic soil properties due to the ice content leads to runoff simulation improvements for both snowmelt- and rainfall-induced floods. No additional parameters were introduced to account for frozen ground effects.

The basic algorithm consists of two main parts. In the first, a physically-based heat transfer component is used to determine the distribution of heat and liquid/frozen water in a soil column. Next, the SAC-SMA model is modified to include the effects of frozen soil on the generation of runoff.

2.1 Heat Transfer Component

Modeling the heat transfer in a column of soil is shown in Figure 1. Two scientific hurdles were overcome before a scheme suitable for the SAC-S MA could be developed. First, the SAC-SMA model does not contain an explicit definition of soil layers. It defines upper and lower tension and free water storages that could be transformed in to a number of soil layers. This hurdle was overcome by recalculating the SAC-SMA storages into their representative soil layers. The SAC-SMA storages, represented as totals of tension and free water, plus water content below wilting point, are recalculated into a required number of soil layers using soil texture data. Three to four layers are usually used with much higher resolution in the upper zone. At each time step, SAC-SMA liquid water storage changes due to rainfall/snowmelt are estimated, and then transformed into soil moisture states of the heat transfer model. The heat transfer model splits the total water content into frozen and liquid water portions based on a simulated soil temperature profile. Estimated new soil moisture states are then converted back into SAC-SMA model storages as shown in Figure 2. The time step of the frozen ground component may be a fraction of the SAC-SMA time step.

The second hurdle to be overcome was that the heat transfer component in Figure 1 requires the depth of snow above the soil surface, and the current version of Snow-17 only computed snow water equivalent. Snow-17 was modified to compute this value. A secondary result of this modification was that the resultant time series of computed snow depth are valuable for the calibration of Snow-17.

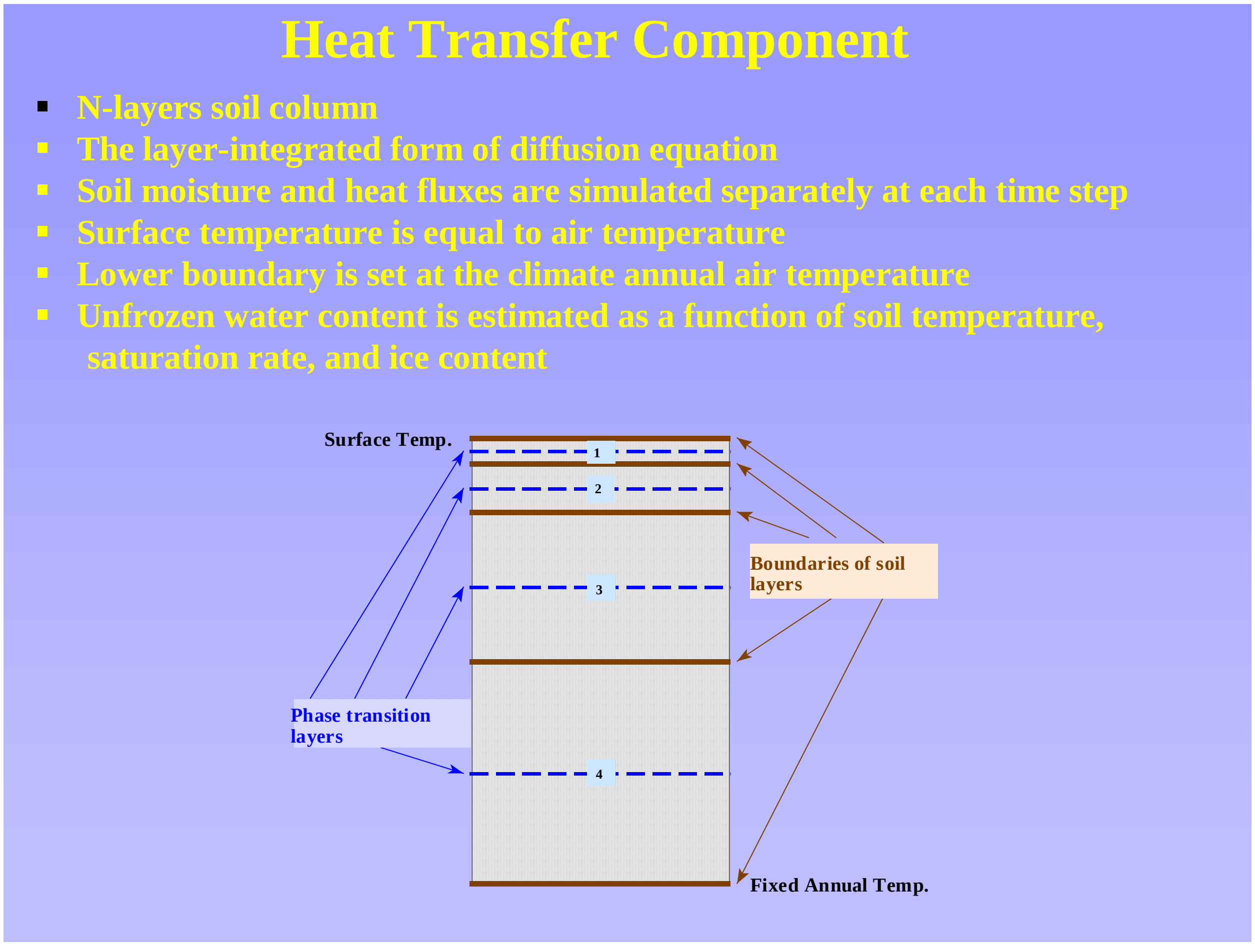


Figure 1. Schematic of the soil layers and heat transfer component.

The parameterization of local processes was based on the heat and moisture transfer equations. The following assumptions were made when using these equations:

1. All water phases are in thermal equilibrium
2. Heat associated with convective water flow can be neglected
3. Liquid water flow in the frozen soil is analogous to that in unfrozen soil
4. The same relations for matric potential and hydraulic conductivity can be used under both frozen and unfrozen conditions.

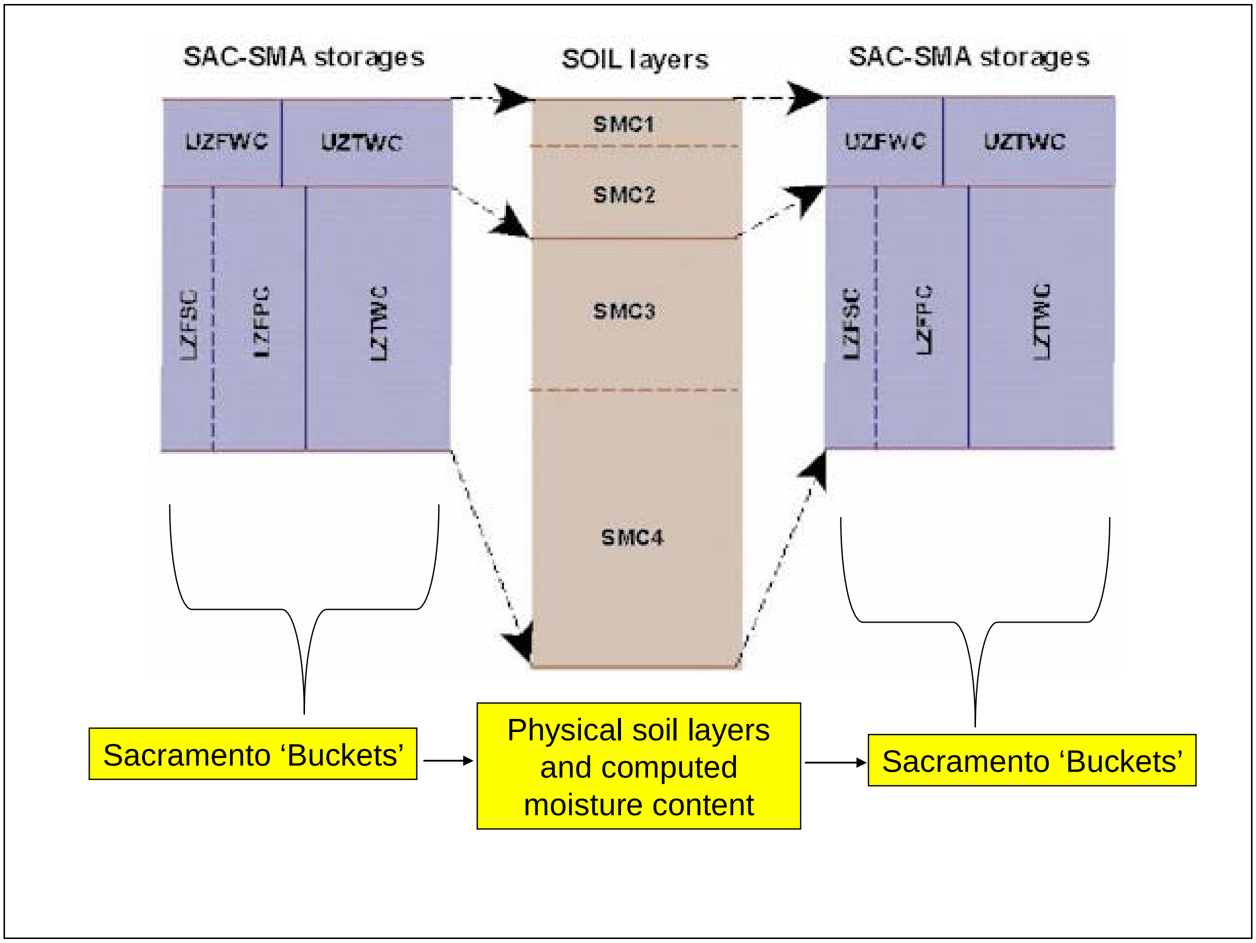


Figure 2. Conversion between Sacramento Model conceptual soil layers and physically based soil layers performed in the Frozen Ground Modification to the Sac Model.

2.2 Modification of SAC-SMA Runoff due to Frozen Water in Soil

In the second part of the algorithm, the SAC-SMA was modified to account for frozen water in the soil calculated by the heat transfer component. The amount of frozen water is then used to adjust the amount of runoff components and percolation computed from the SAC-SMA. The most common formulation of water fluxes under frozen ground conditions (K_{frz}) assumes that soil moisture conductivity (K_o) is reduced depending only on reduction of the liquid water content (W_{liq}):

$$K_{frz} = K_o \left(W_{liq} / W_{sat}\right)^b$$

This first effect on Sacramento water fluxes is accounted by replacing total water storages by liquid water storages only. However, field and experimental data suggest that an increase in the contact surface between solid particles and soil water is a bigger factor. In this formulation, an analytical representation of this effect is derived from Koseni's theory [Kulik, 1969]:

$$K^{*}_{frz} = K_{frz}\,[1 + (a/S_o)W_{ice}]^{-2}$$

Where a/S_o is an increase in the specific surface of solid particles that equals approximately c_k=8. Ratio of the hydraulic conductivity under frozen and non-frozen conditions is applied to the Sacramento interflow and fast ground water runoff components as well as to percolation to the lower zone. It is assumed that the slow ground water component is not affected directly by this factor.

Recent results indicate that the physically-based methodology produces runoff volumes that are as good or superior to those from the calibrated conceptual approach of Anderson and Neuman (1984). In these tests, NCRFC provided input data and SAC - frozen ground (old version) parameters were used. When the new frozen ground model was applied, SAC parameters were adjusted by using an automatic calibration. There was no any adjustment to the frozen ground. Results of these tests are presented in a supporting document 'Frozen Ground White Paper-1.doc'. This is a significant development because the physically-based approach requires no calibration of the frozen ground component while the conceptual formulation specifies seven parameters to be calibrated.

2.3 Effect of spatial variability

In a lumped approach, the model estimates a basin average effect of frozen ground. Because of the large variability of physical properties, the average estimates could be biased. To make the model more flexible, a semi-theoretical treatment of the effect of spatial variability on surface runoff is introduced (Koren, et al., 1999):

$$Y_{surf} = (1 - F_c)Y^{*}_{surf} + F_c P$$

$$F_c = e^{-\alpha \frac{\Theta_c}{\Theta_{ice}}} \sum_{i=1}^{\alpha} \frac{(\alpha \frac{\Theta_c}{\Theta_{ice}})^{\alpha - i}}{\Gamma(\alpha - i + 1)}$$

where Y^{*}_{surf} is surface runoff estimated from averaged frozen ground conditions, Y_{surf} is surface runoff adjusted by a fraction of an impermeable area, F_c, due to frozen ground, P is precipitation or rain+melt, Θ_c is the critical ice content above which the soil is practically impermeable (Koren, 1991), $\alpha=1/C_v$ is a parameter of the gamma distribution of ice content with the coefficient of spatial variation C_v. This effect is calculated in subroutine SURFRZ where Θ_c defined as CRFRZ, and α as CVFRZ (see Section 3.2.1.1.6).

Test results suggest that if frozen ground effects are significant, this variability does not affect much on water balance simulations. Therefore, in most cases, we do not recommend using of this adjustment that requires two not well defined parameters. To exclude this option, a parameter α should be set to any negative value.

2.4 Processing Environment

The Frozen Ground Modification to the SAC-SMA will function in the Calibration, Operational Forecast System (OFS), and the Ensemble Streamflow Prediction (ESP) systems of the NWS River Forecast System (NWSRFS). It requires some soil properties such as texture and bottom boundary soil temperature (this version uses long-term average annual air temperature as a replacement of the boundary condition). During snow cover periods Snow depth in addition to snow water equivalent and snow cover fraction are also required during snow cover periods.

2.5 Parametric Data Needed

The frozen ground modification to the SAC-SMA requires the following parameters for the basin. These are readily available as Hydrologic Rainfall Analysis Project (HRAP) grids from Continental United States (CONUS), and are included in the latest version of the Calibration Assistance Program (CAP).

Soil Texture of the top 0-25 cm layer, and

Long-term mean annual air temperature (soil boundary condition)

2.6 Input Data Needed

2.6.1 Identification.

Snow depth time series defined in NWSRFS data bases as SNSG for simulated or SNOG for observed values.

2.6.2 Acquisition of Inputs

Snow depth time series can be obtained from SNOW-17 operation if the snow depth calculation option is selected. Observed snow depth time series also may be used if available.

3. PROCEDURE

Source code description

3.1. Parameter/State Initialization

NWSRFS basic initialization subroutines are used that are driven by an Input deck. A few new subroutines are added to generate soil profile states and frozen ground parametric information. Initialization process starts from calling subroutine **PIN1:**

3.1.1. PIN1:

Read state and parametric information. To have capability to run the old and new SAC-SMA versions, three run version IDs were defined:

‘FRZE’ – old frozen ground version
‘FREV’ – new frost index, but old calculations of percolation/runoff reduction

‘FRZV’ – new frozen ground version.

Subroutines **FRDFG1** and **FSTFG1** were slightly changed to accommodate addition of the new ground parameter and states.

3.1.1.1. FRDFG1:

Initializes frozen ground parametric information. It first calls subroutine **SOILPAR1** to define soil layer thicknesses.

3.1.1.1.1. SOILPAR1:

Subroutine uses SAC-SMA upper zone and lower zone total water storages (*SUPM=UZTWM+UZFWM* & *SLWM=LZTWM+LZFSM+LZFPM*) as well as soil texture (*STXT*) to split soil profile in a few layers. First, it calculates hydraulic soil properties: porosity (*SMAX*), wilting point (*SWLT*), fraction of quartz (*QUARTZ*), and parameters of the water potential retention curve (*PSISAT & BRT*). Then, it uses this information to estimate soil layer thicknesses by matching a border between upper and lower SAC-SMA zones, and a default soil profile layers (with a higher vertical resolution closer to the soil surface). Outcome of the subroutine is a number of soil layers (*NSOIL*), and their depths as negative values (*ZSOIL*).

The **FRDFG1** subroutine also defines soil temperature states. There are two options:
a) If soil layer depths and temperatures at these depths are available in input deck,
IF(NSINP .GT. 0) THEN
Inputted soil depths will be checked to calculated soil depths by **SOILPAR1** using SAC-SMA parameters: if they are the same, input deck soil temperatures will be assigned to initial states; if they are not equal, input deck soil temperatures will be interpolated into SAC-SMA defined soil profile layers.
b) If soil layer depths and temperatures are not available in input deck,
ELSE
Soil temperature states will be linearly interpolated from soil surface temperature and soil temperature at the lower boundary layer (default value is 2.5m).
ENDIF

3.1.1.2. FSTFG1:

The subroutine stores frozen ground states into PL array, and is basically the same as an original version. If the initial liquid water SAC-SMA states are not available in the input deck, it calls subroutine **ESTFRZ1.**

3.1.1.2.1. ESTFRZ1:

This subroutine initializes liquid water SAC-SMA states. If soil temperature is positive, liquid water states are assigned values of total water SAC-SMA states. If soil temperature is negative, liquid water states are calculated using SAC-SMA total water states and a water potential equilibrium law, Section 3.2, by calling subroutines **TBND, ST_AVG1**, and **FRH2O** described in the next section.

Initialization of liquid and total soil moisture states of the soil profile of the frozen ground component is performed in **EX1** subroutine by calling subroutine:

3.1.2. EX1:

3.1.2.1 FRZINIT1:
The subroutine distributes uniformly SAC-SMA total water contents of upper and lower zones. It calculates potential liquid water contents at each soil layer from a water potential equilibrium law, Section 3.2, using soil temperature states. Then, it checks an agreement of estimated potential liquid water contents and SAC-SMA liquid water states, and adjusts soil layer states if needed to keep water balance.

3.2 Simulation Component

Simulation process starts from Subroutine **FLAND1,** a basic Sacramento model changed to account for the new frozen ground version.

3.2.1 FLAND1:

The original SAC-SMA code structure was not changed. However, new liquid water states are added, **UZTWH, UZFWH, LZTWH, LZFSH, and LZFPH**, in addition to the total water storages. As a result, some new statements were added to perform:

a) calculation of dynamics of the new liquid water states. It is assumed that only liquid water is movable;
b) simulation of reductions of interflow, fast ground water runoff, and percolation due to frozen ground, as described in Section 3.2;
c) adjustment of surface runoff due to spatial variability of ice content, and as a result, potential impermeable area;
d) calculation of dynamics of the heat transfer states, soil temperature, frozen and liquid soil moisture, in the soil column, as described in Section 3.2. So, it calls the main frozen ground Subroutine **FROST2_1.**

3.2.1.1 FROST2_1:
It calls a number of Subroutines to perform calculations that are follows in the sequence:

3.2.1.1.1 SAC2FRZ1:
Recalculate SAC states into soil profile states. Free water is distributed equally. Tension water is distributed differently: if it increased from the

previous time interval, it is distributed by the ratio of ice content; if it decreased, it is distributed by the inverse ratio of total water deficit.

3.2.1.1.2 HRT1:
HRT1 performs heat transfer simulation over the soil profile. Simulation time increment for the heat transfer could be a fraction of the Sacramento time step. Default time increment is set to 30 min. The subroutine actually generates a matrix of coefficients of the heat transfer numerical equation by calling a number of Subroutines:

3.2.1.1.2.1 CND_JOHNS:
Computes thermal soil conductivity under frozen ground conditions using Johnson's method.

3.2.1.1.2.2 CSNOW:
Computes snow conductivity from snow density using Dychkova equation.

3.2.1.1.2.3 TBND:
Calculate soil layer boundary temperature from soil profile temperature states.

3.2.1.1.2.4 SNKSRC:
Calculates sink/source term of heat fluxes. Calls:

3.2.1.1.2.4.1 ST_AVG1:
Estimates an average soil layer temperature.

3.2.1.1.2.4.2 FRH2O:
Calculates the liquid water fraction of a soil layer.

3.2.1.1.3 HSTEP:
Estimates soil temperature and frozen soil moisture states for the soil profile using a numerical matrix from the subroutine **HRT1**. It calls subroutine **ROSR12.**

3.2.1.1.3.1 ROSR12: Solve the tri-diagonal matrix.

3.2.1.1.4 FRZ2SAC1:
Transforms soil profile states into SAC states by calling Subroutine **FST2SAC1** twice for the upper and lower zones.

3.2.1.1.4.1 FST2SAC1:
Calculates SAC states for the selected zone. Calculations are based on a liquid water change per time interval.

3.2.1.1.5 FRZIND1:
Calculates a frost index, replication of the old one. It is used only if surface runoff adjustment due to impermeable areas is activated. This index also can be used to compare old and new frost indexes.

3.2.1.1.6 SOIL_INT1:
Interpolates soil profile states into desired regular soil profile layers. They can be used for diagnosis, e.g., compare to measured soil moisture at different soil

layers.

Adjust runoff due to the frozen ground:

a) **IF(version 'FRZE' or 'FREV')**
CALL FGFR1():
The original version frozen ground treatment.

b) **IF(version 'FRZV' and 'Critical ice content, parameter CRFRZ' > 0)**
CALL SURFRZ1(FROST,CVFRZ,CRFRZ):
3.2.1.2 SURFRZ1:
The subroutine adjusts surface runoff due to a potential impermeable area because of spatial variability of soil ice content, see section 1.1. This subroutine is called only if a parameter of critical ice content is defined. In most cases, we do not recommend the use of this component.

4. COMPUTATION

4.1 Notation

c	=	volumetric heat capacity
ρ	=	water density
L	=	latent heat of fusion
Δz	=	soil layer thickness
K	=	thermal conductivity
T	=	soil temperature
Θ_{ice}	=	ice content
Θ	=	total water content
Θ_s	=	soil porosity
Ψ	=	soil water potential at saturation

4.2 Symbolic Formulas

COMPUTE : layer integrated diffusion equation

$$c\Delta z_i \frac{\partial T_i}{\partial t} = (K\frac{\partial T}{\partial z})_{z_{i+1}} - (K\frac{\partial T}{\partial z})_{z_i} + \rho L \Delta z_i \frac{\partial \Theta_{ice,i}}{\partial t}$$

COMPUTE liquid water content depending on soil temperature, total and frozen water

$$\frac{g\Psi_s}{L}(1+c_k\Theta_{ice})^2(\frac{\Theta-\Theta_{ice}}{\Theta_s})^{-b} - \frac{T}{T+273.16} = 0$$

COMPUTE the depletion fractions of free water storages to calculate interflow and fast lower zone runoff due to frozen water in the soil:

$DUZ = 1 - [1 - UZK * (1 + c_k W_{ice})^{-2}]^{DINC}$

$$DLZS = 1 - [1 - LZSK * (1 + c_k W_{ice})^{-2}]^{DINC}$$

and percolation reduction, $PERC_{frz}$

$$PERC_{frz} = PERC * (1 + c_k W_{ice})^{-2}$$

- *DUZ* and *DLZS* are depletion fractions of interflow and fast runoff respectively,
- *UZK* and *LZSK* are SAC parameters,
- *DINC* is a time increment of percolation calculations (original SAC code),
- *PERC* is percolation under non-frozen ground conditions.

APPENDICES

Appendix A Acronym List

AEL Algorithm Enunciation Language
AP Anomalous Propagation
AWIPS Advanced Weather Interactive Processing System
CAP Common Alerting Protocol
CONOPS Concept of Operations
DHR Digital Hybrid Scan Reflectivity
DPA Hourly Digital Precip Array
DSP Digital Storm Total Precip
ECP Engineering Change Proposal
HL Hydrology Laboratory
HSEB Hydrologic Software Engineering Branch
HSMB Hydrologic Science and Modeling Branch
HSR Hybrid Scan Reflectivity
NCDC National Climatic Data Center
NEXRAD NEXt Generation RADar
NOAA National Oceanic and Atmospheric Administration
NWS National Weather Service
NWSRFS National Weather Service River Forecast System
OHD Office of Hydrologic Development
OHP One Hour Precip (1 Hr Surface Rainfall Accumulation)
RDA Radar Data Acquisition
RFCs River Forecast Centers
ROC Radar Operations Center
RPG Radar Products Generator
SAC Sacramento Model Soil Moisture Accounting Model
SAC-SMA Sacramento Model Soil Moisture Accounting Model
SON Statement of Need
VCP Volume Coverage Pattern
WFOs Weather Forecast Offices

Appendix B Scientific Description Addendum

B.1. Scientific Description

Heat and moisture transfer processes in the aeration zone play an important role in the runoff generation mechanism in regions where seasonal soil freezing/thawing occurs. NWSRFS has a conceptual modification to the Sacramento Soil Moisture Accounting Model (SAC-SMA) that simulates a frost index and makes SAC-SMA runoff adjustment depending on this index. As a conceptual model, this approach requires the calibration of 7 parameters.

Recent developments in land surface modeling have significantly improved representation of cold season processes. However, a conceptual representation of a soil profile in commonly used watershed models complicates implementation of physically-based heat-moisture transfer models that require numerical integration over the soil profile. Another challenge is the formulation of the effects of frozen ground on water fluxes, specifically the partitioning of meltwater/rainfall into surface runoff and infiltration. In watershed modeling, simplified approaches are typically used such as empirical equations of the percolation reduction or water balance-type approaches. The latter assumes the percolation reduction depends only on frozen water-induced soil porosity reduction. However, field and experimental data suggest that an increase in the surface contact between solid particles and soil water may also be a factor.

Capitalizing on successful collaboration between the Office of Hydrologic Development and National Center for Environmental Prediction (NCEP), the Hydrology Laboratory (HL) developed a physically-based parameterization that addresses two of the problems mentioned above: 1) modification of a storage-type SAC-SMA model to be compatible with the theoretical heat transfer model, and 2) parameterization of frozen ground effects on runoff derived from Kozeny's theory that accounts for changes in both solid particles-water surface contact and 'free' porosity.

The basic algorithm consists of two main parts. In the first, a physically-based heat transfer component is used to determine the distribution of heat and liquid/frozen water in a soil column. Next, the SAC-SMA model is modified to include the effects of frozen soil on the generation of runoff. A conceptual representation of soil moisture fluxes combined with a physically-based heat transfer model provides reasonable simulations of water and heat exchanges over a soil profile. Physically-based modification of the hydraulic soil properties due to the ice content leads to runoff simulation improvements for both snowmelt- and rainfall-induced floods. No additional parameters were introduced to account for frozen ground effects.

B.1.1 Methodology or Theory – (equations and computations)

B. 1.1.1. Combining heat-moisture transfer components

The SAC-SMA model is used to estimate soil moisture states and runoff components, and a layer integrated form of the heat transfer model (Koren et al., 1999) is used to estimate soil temperature and unfrozen water states. The SAC-SMA model consists of upper and lower tension and free water storages that interact to generate five runoff components. Koren et al. (2003) have developed a set of relationships that link the SAC-SMA storages (parameters) and soil properties such as porosity, field capacity, wilting point, and hydraulic conductivity. They assume that tension water storages of the SAC-SMA model are related to available soil water, and that free water storages are related to gravitational soil water.

The model parameter-soil property relationships allow recalculating the upper and lower soil moisture capacities into soil moisture contents at a number of soil layers. Five layer depths are defined a priori to cover a 2 m soil profile with thinner layers closer to the soil surface. However, an actual number of soil layers and their thicknesses are automatically adjusted using SAC-SMA parameter values at the selected location. To make this adjustment, the upper, Z_U, and lower, Z_L, zone depths are estimated first to be sure that the upper and lower SAC-SMA capacities are preserved (for the schematic of SAC-SMA storages and heat transfer layers interaction see **Fig. 1**):

$$Z_U = \frac{UZTWM + UZFWM}{\theta_s - \theta_{wlt}} \tag{1}$$

$$Z_L = Z_U + \frac{LZTWM + LZFSM + LZFPM}{\theta_s - \theta_{wlt}} \tag{2}$$

where ***UZTWM, UZFWM, LZTWM, LZFSM***, and ***LZFPM*** are SAC-SMA parameters: upper zone tension and free water, and lower zone tension water and supplemental and primary free water storages respectively, θ_s and θ_{wlt} are saturation soil moisture content and wilting point respectively. A priori defined layer depths are then adjusted to be consistent with the Z_U and Z_L estimates. Because of this, the number of soil layers may be less than five, and can be different for different pixels/basins. A variable number of soil layers can be used in the heat transfer model (Koren et al., 1999) simplifying the coupling to the SAC-SMA model. To be compatible with SAC-SMA complexity, e.g. the use of precipitation and potential evaporation input data only, the soil/snow surface heat balance calculation of the original heat transfer model is replaced by applying air temperature as a soil surface temperature. At each time step, SAC-SMA liquid water storage changes due to snowmelt/rainfall are estimated, and then they are transformed into the layered soil moisture states of the heat transfer model. The heat transfer model splits the total water content into frozen and liquid water portions based on simulated soil temperature profile. Newly estimated soil moisture states are then converted back into the SAC-SMA model storages using the same relationships between SAC-SMA storage parameters and soil properties as described above.

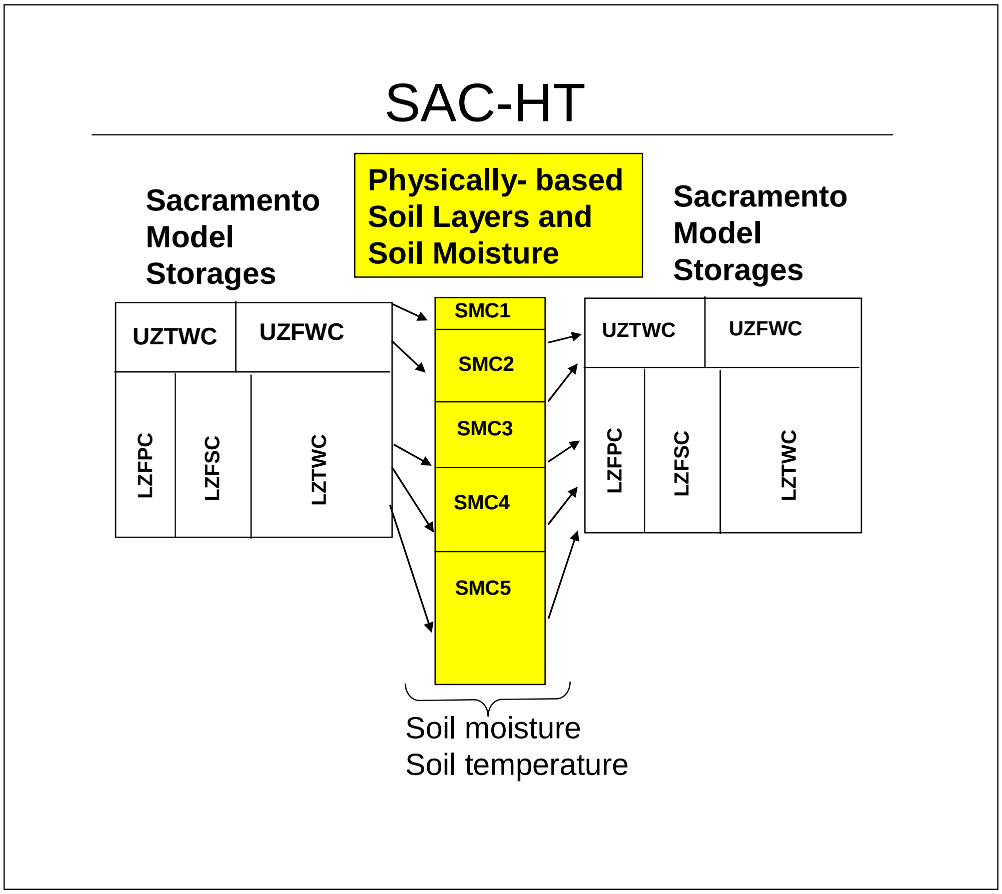


Fig. 1. Conversion between Sacramento Model conceptual soil layers and physically based soil layers performed in the Frozen Ground Modification to the Sac Model.

The basic heat transfer model equations are used in this formulation. Layer integrated diffusion equation:

$$c\Delta z_i \frac{\partial T_i}{\partial t} = (K\frac{\partial T}{\partial z})_{z_{i+1}} - (K\frac{\partial T}{\partial z})_{z_i} + \rho L \Delta z_i \frac{\partial \Theta_{ice,i}}{\partial t} \qquad \textbf{(3)}$$

Liquid water content depending on soil temperature, total and frozen water was estimated from the Clausius-Clapeyron equation for phase equilibrium combined with the Campbell's relationship between water potential and water content adjusted for the effects of frozen soil:

$$\frac{g\Psi_s}{L}(1+8\Theta_{ice})^2(\frac{\Theta-\Theta_{ice}}{\Theta_s})^{-b} - \frac{T}{T+273.16} = 0 \qquad \textbf{(4)}$$

Following notations were used in Eqs. (3) and (4): ***c*** is the volumetric heat capacity, **ρ** is the water density, ***L*** is the latent heat of fusion, ***Δz*** is the soil layer thickness, ***K*** is the thermal conductivity, ***T*** is the soil temperature, $\boldsymbol{\Theta_{ice}}$ is the ice content, **Θ** is the total water content, $\boldsymbol{\Theta_s}$ is the soil saturation, **Ψ** is the soil water potential at saturation.

B.1.1.2. Formulation of water fluxes due to frozen ground effects

The most common formulation of water fluxes under frozen ground conditions assumes that reduction of soil moisture conductivity depends only on the ratio of a liquid water content (θ_l) to the soil saturation (θ_s), e.g.:

$$K_f = K_0 f(\frac{\theta_l}{\theta_s}) \quad \textbf{(5)}$$

In this formulation, liquid water content is used instead of total water content but the saturated hydraulic conductivity, K_0, is assumed to be the same value as for the unfrozen soil. However, field and experimental data suggest that an increase in the contact surface between solid particles and soil water is a bigger factor. To account for this additional frozen ground effect on the saturated hydraulic conductivity K_0, we use Kozeny's theory that relates the filtration rate and the soil particles – water contact surface, S_0:

$$K_0 = A\frac{P^3}{S_0^{\ 2}} \quad \textbf{(6)}$$

where P is the soil porosity, and A is a parameter that depends on soil properties.

Following Kulik (1969), we assume that increase in the contact surface due to ice crystals equals $a\theta_f$, where a is an increase in the ice crystal surface per unit of ice content, and θ_f is the ice content. By including this additional contact surface in Eq. (6) and dividing resulted equation by (6), one can obtain an analytical formulation of this effect on the saturated hydraulic conductivity of the frozen ground:

$$K_0^* = K_0 \frac{1}{[1+(a/S_0)\theta_f]^2} \quad \textbf{(7)}$$

and

$$K_f = K_0^* f(\frac{\theta_l}{\theta_s}) \quad \textbf{(8)}$$

It can be seen from equations (7) and (8), that the hydraulic conductivity can be reduced considerably due to the change in the surface contact in addition to the liquid water ratio. Experimental data suggest that the value of a/S_0 does not vary significantly for different soils, and a value of 8 is its reasonable estimate.

The original SAC-SMA model runoff components are formulated as linear reservoirs with constant withdrawel rates which can be expressed as a linear function of the hydraulic conductivity. Therefore, to account for the frozen ground effect, the ratio of the hydraulic conductivity from (7) to the saturated hydraulic conductivity K_0 is applied to the interflow and fast ground water runoff components as well as to percolation to the lower zone. It is assumed that the slow ground water component from a deeper soil layer is not affected by this factor directly. As a result, the original SAC-SMA runoff rates are rewritten as:

$$DUZ = 1 - [1 - UZK * (1 + 8*W_{ice})^{-2}]^{DINC} \quad \textbf{(9)}$$

$$DLZS = 1 - [1 - LZSK * (1 + 8*W_{ice})^{-2}]^{DINC} \quad \textbf{(10)}$$

and percolation reduction, $PERC_{frz}$

$$PERC_{frz} = PERC * (1 + 8*W_{ice})^{-2} \qquad (11)$$

- ***DUZ*** and ***DLZS*** are depletion fractions of interflow and fast runoff respectively,
- ***UZK*** and ***LZSK*** are SAC-SMA parameters,
- ***DINC*** is a time increment of percolation calculations (original SAC-SMA code),
- ***PERC*** is percolation under non-frozen ground conditions.

B.2 Application results

Different output results from the parameterization were analyzed in a number of applications.

Simulated soil temperature at a number of operational stations over the northwestern part of the US was compared with available measurements. At most stations, soil temperatures were measured at multiple depths, 5, 10, 20, 50, and 100 centimeters once per day. Model parameters are estimated from soil-vegetation data (Koren et al., 2003) without further calibration. Daily frozen and liquid water contents and soil temperature at five layers are simulated for 3-5 years. Overall simulated soil temperature dynamics are represented well for all 5 soil layers. Correlation coefficients between simulated and measured soil temperatures are above 0.95 while there is a slight decrease in correlation for the top layer; see Table 1 for statistics at three selected soil layers. Some reduction in variability of the top layer soil temperature can be explained by the use of daily input data. Root mean square errors and Nash-Sutcliffe efficiency (Nash and Sutcliffe, 1970) are also better for the deeper soil layer.

Similar simulations were performed for the Valdai, Russia experimental station, where soil moisture measurements were available at three soil layers. Simulated soil temperature and soil moisture agree well with measurements over the eight years period, **Fig. 2**.

Table 1. Accuracy statistics of simulated soil temperature at three layers: root mean square error (RMS), Nash-Sutcliffe efficiency (NS), and correlation coefficient (R)

Station name	NCDC	5 cm layer			20 cm layer			50 cm layer		
	Site ID	RMS	NS	R	RMS	NS	R	RMS	NS	R
Lamoni, IA	134585	3.0	0.91	0.96	2.9	0.90	0.99	2.9	0.88	0.99
Atlantic, IA	130364	5.0	0.77	0.96	3.3	0.86	0.99	2.9	0.82	0.98
Burlington, IA	131060	3.3	0.84	0.96	2.5	0.88	0.97	3.0	0.79	0.97
Des Moines, IA	132209	3.7	0.89	0.97	1.9	0.95	0.99	1.6	0.95	0.99
Estherville,IA	132724	4.5	0.83	0.96	2.0	0.95	0.98	2.6	0.88	0.98
Toledo, IA	138296	3.2	0.89	0.97	3.2	0.88	0.98	2.3	0.91	0.99
Preston, MN	216654	3.9	0.87	0.97	2.4	0.91	0.98	2.4	0.87	0.99
Waubay, SD	398980	3.3	0.90	0.96	1.5	0.97	0.99	1.3	0.97	0.99
Spencer, IA	137844	8.1	0.67	0.95	5.1	0.80	0.98	3.6	0.85	0.99
Mean		4.2	0.84	0.96	2.8	0.90	0.98	2.5	0.88	0.99

Runoff simulation tests were performed for a number of river basins in Minnesota by OHD and NCRFC. 6-hourly precipitation and air temperature averaged over each basin and daily discharges at basin outlets are available for a 25-40 year period. Empirical unit hydrographs are used to transform basin average simulated runoff into outlet hydrographs. First, we calibrate SAC-SMA parameters of frozen and non-frozen ground versions using 10 years of data. Note, that the frozen ground version does not have any additional parameters to calibrate. The remaining 15-30 years of data are used for test purposes. Root mean square errors of daily and monthly discharges for the test period are shown in Tables 2 and 3. The frozen ground version outperforms the non-frozen ground version for all basins. For a number of basins, spring floods are significantly underestimated and early summer floods are overestimated from the non-frozen version. These results suggest that the model parameter calibration can not fix the non-frozen ground version physics problem in such cases.

Table 2. Comparison of simulation statistics from the original and new frozen ground models. (OHD test basins)

Statistics	1 Old FRZN	2 New FRZN, adjusted SAC	3 New FRZN, un-adj. SAC	4 Old FRZN	5 New version, Adjusted SAC	6 New version, un-adj SAC
	LNEM5 (Root River) Calibrated by Eric Anderson			GRDM5 (Watonwan River)		
Daily RMS, cms	14.67	13.40	14.49	11.57	10.30	11.68
Daily ABS, cms	4.62	4.10	4.63	6.25	5.42	6.22
Monthly ABS, mm	3.84	3.72	4.11	6.35	5.39	6.22
Monthly RMS, mm	5.79	5.68	6.04	9.20	7.80	9.19
Bias, %	7.75	0.71	10.70	-24.80	-6.13	-22.56
Correlation, R	0.73	0.72	0.73	0.84	0.85	0.83
	MMLM5 (Redwood River)			DBCN8 (Baldhill Creek)		
Daily RMS, cms	3.34	3.26	3.63	2.11	2.14	2.47
Daily ABS, cms	1.24	1.24	1.29	0.52	0.54	0.56
Monthly ABS, mm	4.05	4.07	4.31	1.17	1.23	1.22
Monthly RMS, mm	4.67	8.06	8.77	3.38	3.23	3.48
Bias, %	1.78	7.14	5.23	-14.50	-2.95	-3.62
Correlation, R	0.83	0.84	0.80	0.78	0.77	0.73

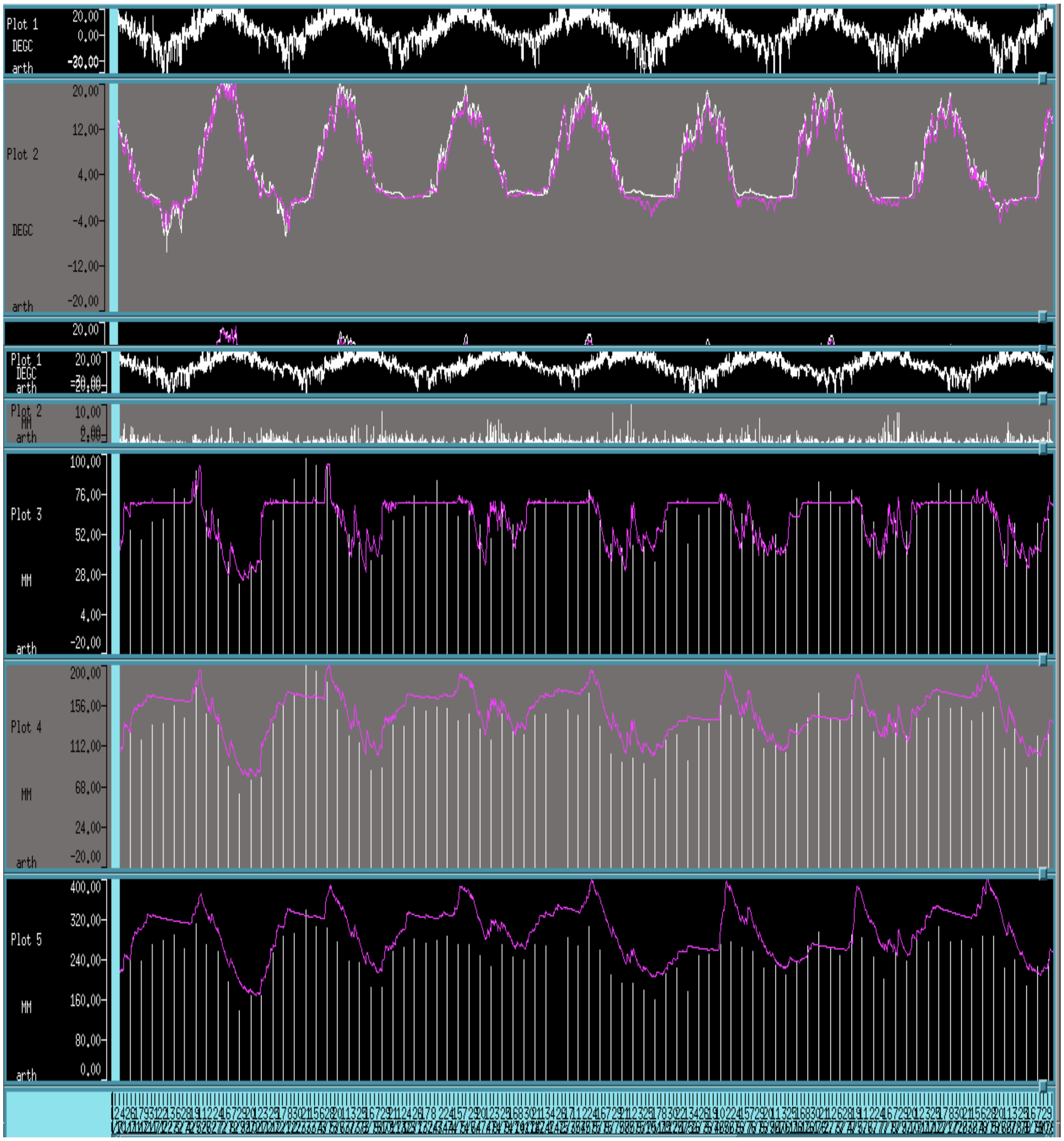
Notes:
1. Old FRZN and New FRZN with SAC adjusted parameters are colored in blue if differences were more than 2.5% (Columns x and z)
2. Statisitics for New FRZN with un-adjusted SAC parameters are colored in red if worse than 2.5% (compare columns 1 to 3 and 4 to 6)
3. Columns 1, 4. Model parameters obtained from NCRFC or in one case (Root River) Eric Anderson's calibration. It is not known how NCRFC calibrated the Sac model in conjunction with the Old FRZN model. It could be that the SAC model was inappropriately calibrated to match frozen ground events
4. Columns 2, 5. Use New FRZN and adjust the SAC model parameters.
5. Columns 3. 6. Use New FRZN and NCRFC parameters.

Table 3. Comparison of simulation statistics from the original and new frozen ground models (NCRFC test basins)

Statistics	1 Old FRZN	2 New FRZN, adjusted	3 New FRZN, un-adj. SAC	4 Old FRZN	5 New FRZN, adjusted SAC	6 New FRZN, Un-adj. SAC
	RPDM5 (Rapidan, MN)			**LQPM5 (Parle, MN)**		
Daily RMS, cms	17.60	17.29	18.60	8.73	8.32	9.10
Daily ABS, cms	8.82	8.49	8.91	3.42	2.81	3.66
Monthly ABS, mm	6.88	6.46	6.97	2.92	2.34	3.15
Monthly RMS, mm	10.50	9.46	10.80	5.30	4.65	5.58
Bias, %	3.31	-8.88	7.42	25.60	1.92	38.40
Correlation, R	0.83	0.84	0.81	0.86	0.86	0.87
	LGPM5 (Long Prairie, MN)			**AMEN8 (Amenia, ND)**		
Daily RMS, cms	3.63	3.34	3.90	1.83	1.57	1.78
Daily ABS, cms	2.05	2.04	2.12	0.39	0.37	0.39
Monthly ABS, mm	4.26	4.21	4.39	2.88	2.76	2.88
Monthly RMS, mm	6.06	5.96	6.40	7.08	5.99	7.19
Bias, %	20.60	22.03	22.10	31.90	32.24	38.40
Correlation, R	0.82	0.84	0.81	0.61	0.67	0.64

Notes:
1. Old FRZN and New FRZN with SAC adjusted parameters are colored in blue if differences between Columns 1 and 2 were more than 2.5%
2. New FRZN with un-adjusted SAC parameters are colored in red if statistics were worse than 2.5% (compare columns 1 to 3 and 4 to 6)
3. Columns 1, 4. Model parameters obtained from NCRFC. It is not known how NCRFC calibrated the Sac model in conjunction with the Old FRZN. It could be that the SAC model was inappropriately calibrated to match frozen ground events
4. Columns 2, 5. Use New FRZN and adjust the SAC model parameters.
5. Columns 3, 6. Use New FRZN and NCRFC parameters. to replicate the tests that led to NCRFC comments on the HOSIP documents

Fig. 2. Observed (white) and simulated (red) soil temperature and moisture at 20cm, 40cm, and 80cm depths. Valdai, Russia, 1971-1978.



B.3 Limitations

Interaction between the conceptual SAC-SMA storages and the heat transfer soil layers is defined based on soil properties which can vary significantly over a river basin. This can lead to the degradation of simulation results for river basins with significant variability in soil properties. To improve results in such cases, a basin should be broken into a number of quasi-uniform sub-basins.

B.4 Future Work

This algorithm was recently incorporated into the Hydrology Laboratory Research Distributed Hydrologic Model (HL-RDHM) and tested over large areas. Use of HL-RDHM will reduce the potential degradation of simulation results for basins affected by significant variability of soil properties.