Using Wavelet Analysis to Assess Timing Errors in Streamflow Predictions

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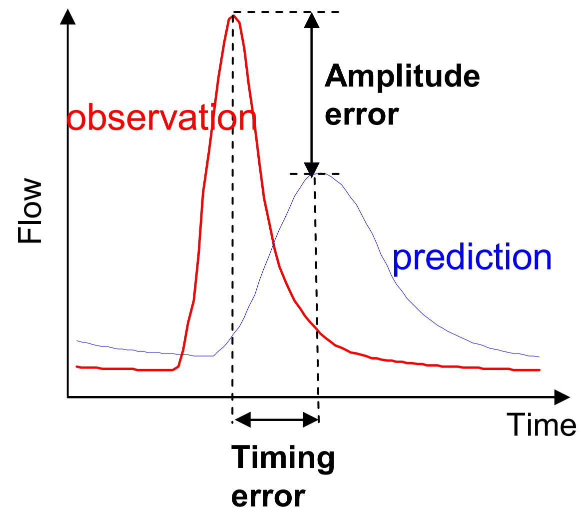
Timing Error: What & Why

Timing Error

The displacement (in time) of a distinct hydrologic feature (e.g., flow peaks) in streamflow predictions



Why Timing Error Assessment

Timing & amplitude errors are different by nature

Distinct information on timing & amplitude uncertainty is important for:

Model evaluation: more meaningful error statistics

Model diagnosis: distinct sources of uncertainty

Decision support: effective responses, efficient uses of resources

Assessing Timing Errors

Challenges

Need to account for **scale- & time-dependencies** of timing errors

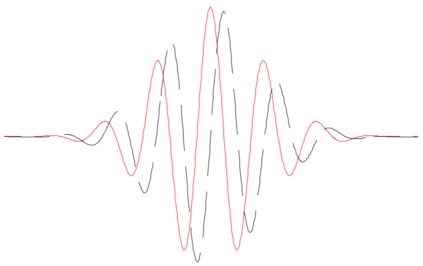
Need to identify **distinct features**, e.g., event peaks

Need **sufficient data** to reduce sampling uncertainty

Potential Approaches

Wavelet analysis

- ✓ **Cross wavelet transform** for timing error analysis
- wavelet transform for event identification



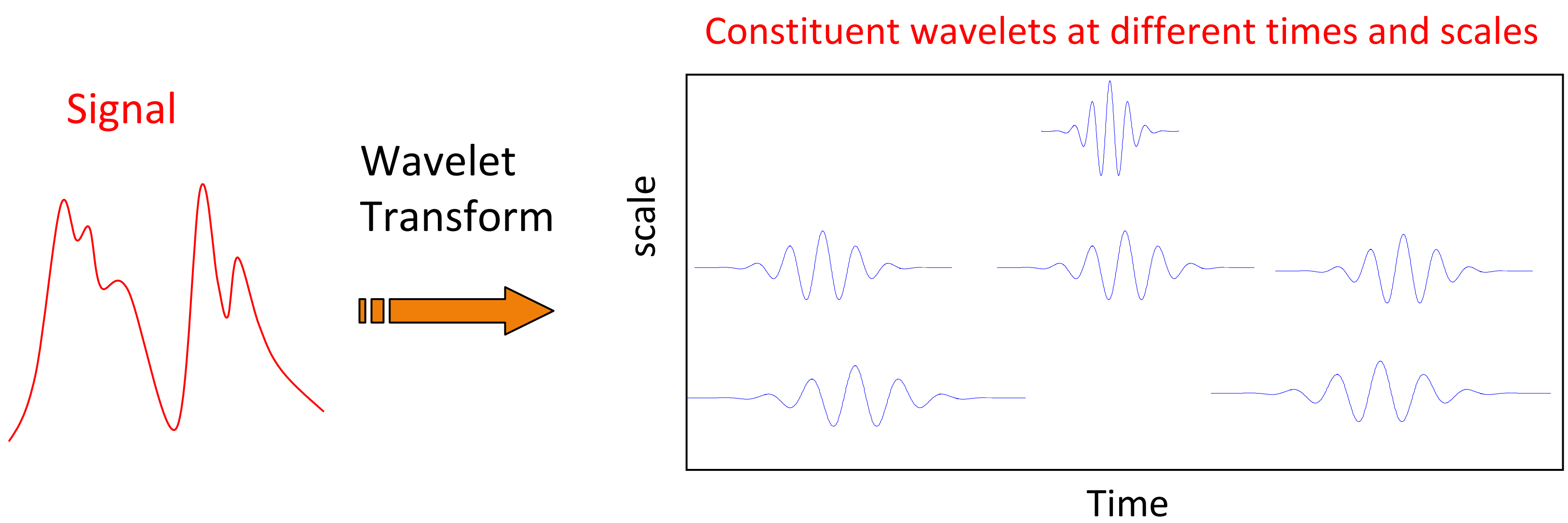
Other approaches

- Curve registration, landmark registration
- Dynamic time warping
- Windowed Fourier analysis?

Continuous Wavelet Transform (CWT)

Convolution of a discrete sequence *x(t)* with a scaled and translated version of a mother wavelet ψ_0

$$W(scale, position) = \int_{-\infty}^{\infty} x(t)\psi_0(scale, position)dt$$



Cross Wavelet Transform (XWT)

- XWT of two time series Z_1 and Z_2:

$$W^{Z_1Z_2} = W^{Z_1} W^{Z_2 *}$$

CWT of Z_1 — CWT of Z_2 — * complex conjugate

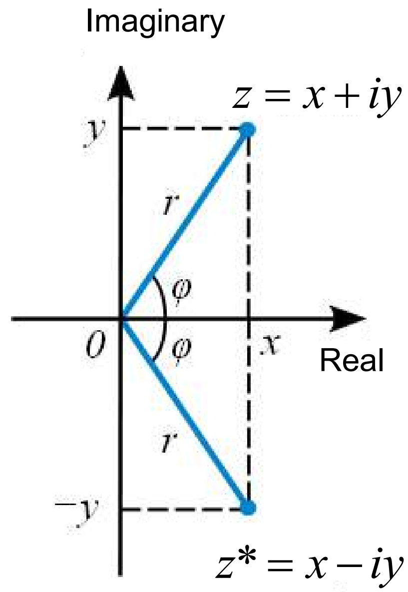
- Cross wavelet power $\left| W^{Z_1Z_2} \right|$

- Cross Wavelet coherence $\dfrac{\left| W^{Z_1Z_2} \right|}{\left| W^{Z1} \right| \left| W^{Z2} \right|}$

- Phase relationship $\varphi = \arg(W^{Z_1Z_2})$

Timing error $\varphi * T / \pi$

T: period of wavelet



Choosing a Mother Wavelet

Factors to consider

- Continuous or discrete
- Real or complex
- Width of wavelet function
- Shape of wavelet function

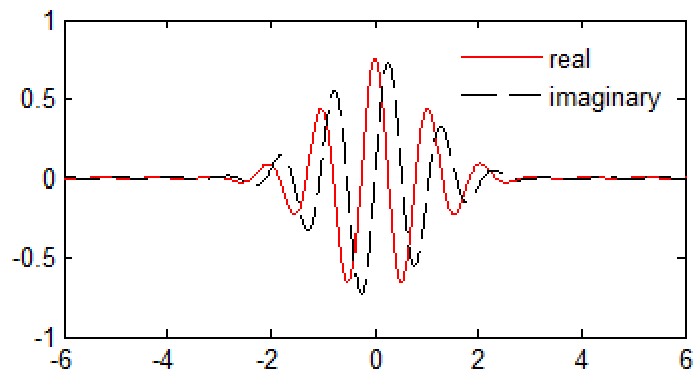
Complex Morlet Wavelet

Widely used

Good balance between time & frequency resolution

$$\psi_0(\eta) = \pi^{-1/4} e^{iw_0\eta} e^{-\eta^2/2}$$

$$(\omega_0 = 6)$$



Test Basins

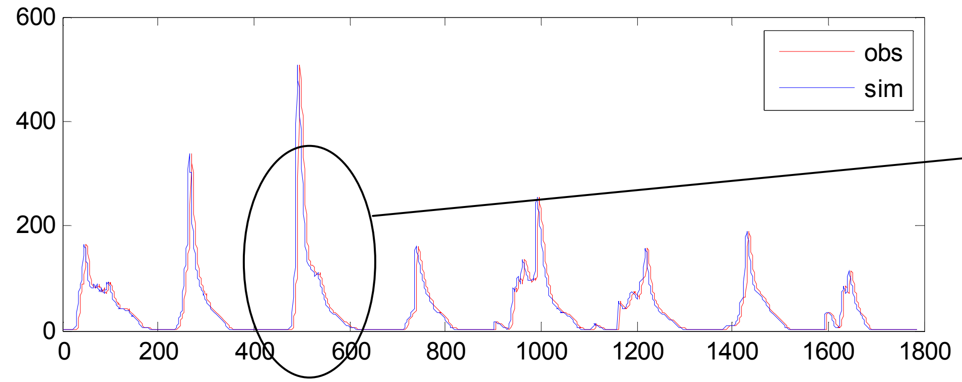
The 11 headwater study basins in Texas

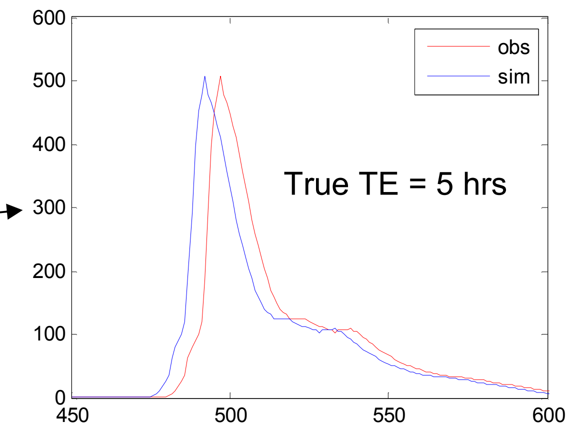
	Basin	Area (km^2)	Annual Precipitation (mm)	Time to Peak (hr)
1	QLAT2	197	793	12
2	SDAT2	285	1103	17
3	MCKT2	427	800	14
4	MTPT2	435	1126	17
5	LYNT2	508	858	18
6	SKMT2	640	712	12
7	MDST2	870	933	21
8	SBMT2	896	934	26
9	SCDT2	932	845	14
10	REFT2	1787	748	39
11	UVAT2	1981	515	13

~ 10 years of hourly streamflow observation and simulation data in each basin

Testing XWT on Synthetic Timing Error

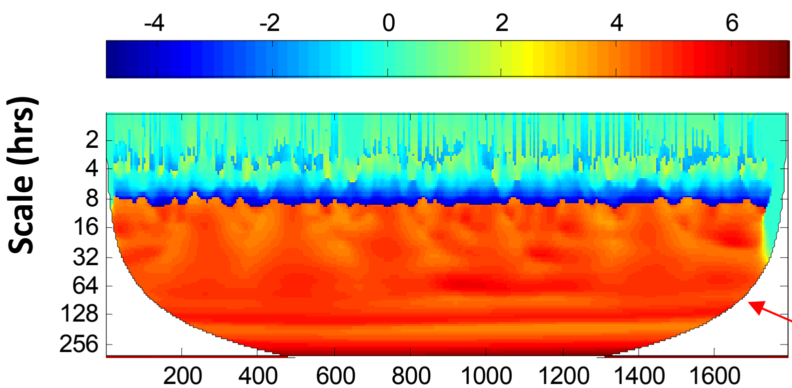
Observation & Synthetic Simulation





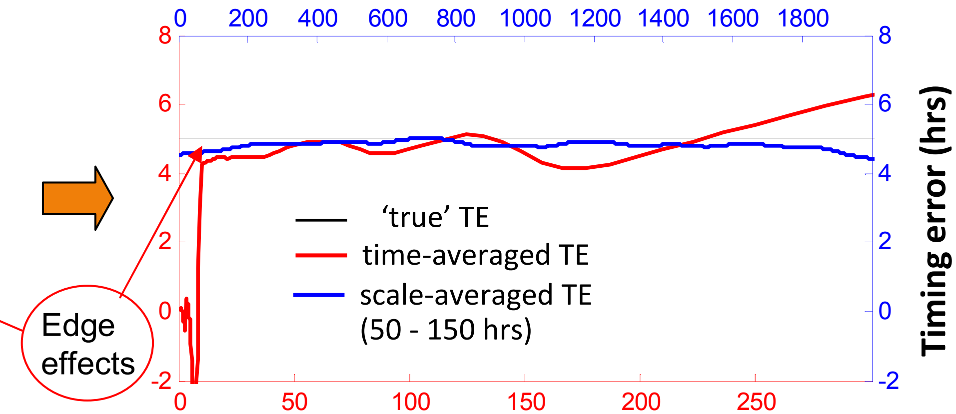
XWT

Timing Error Spectrum



Time (hrs)

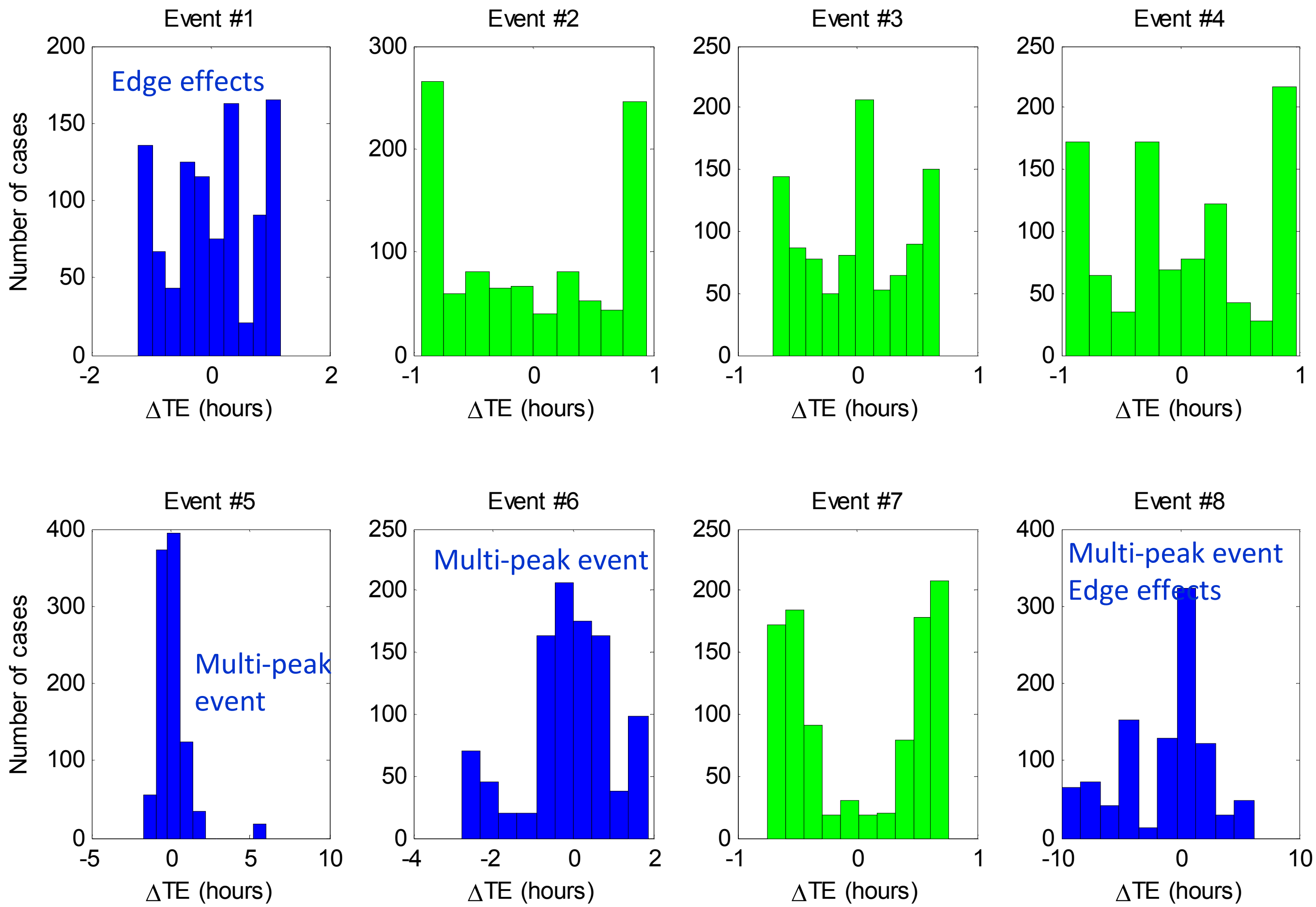
Time (hrs)



Scale (hrs)

Testing with Monte Carlo Simulations of Synthetic TE

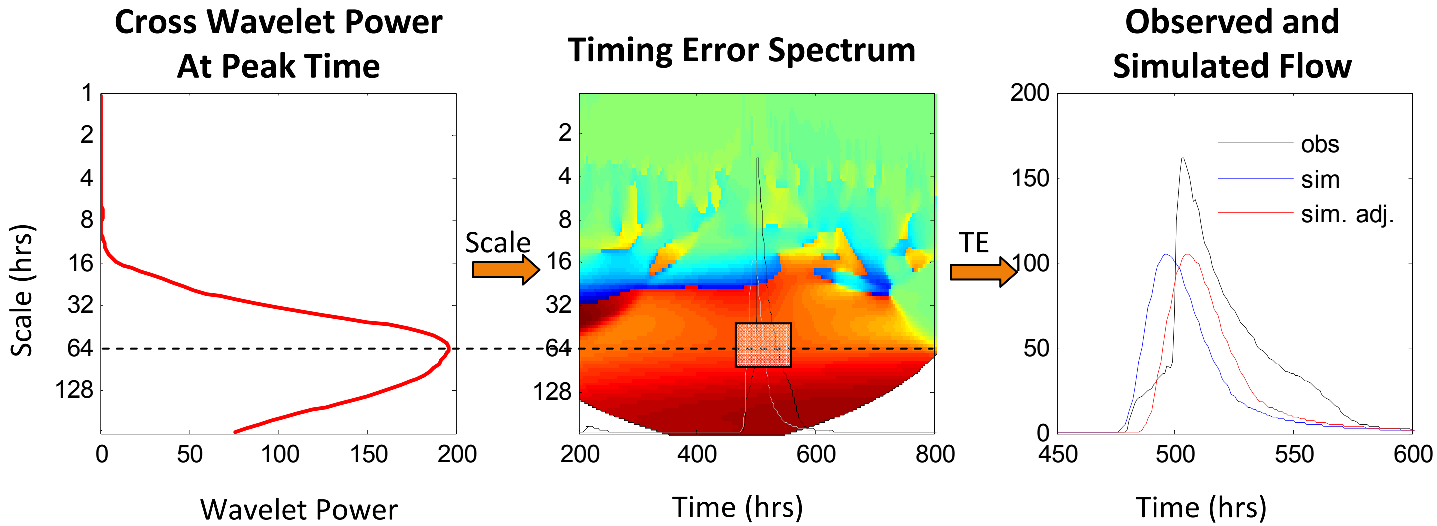
Histograms of ΔT (Estimated TE – True TE)

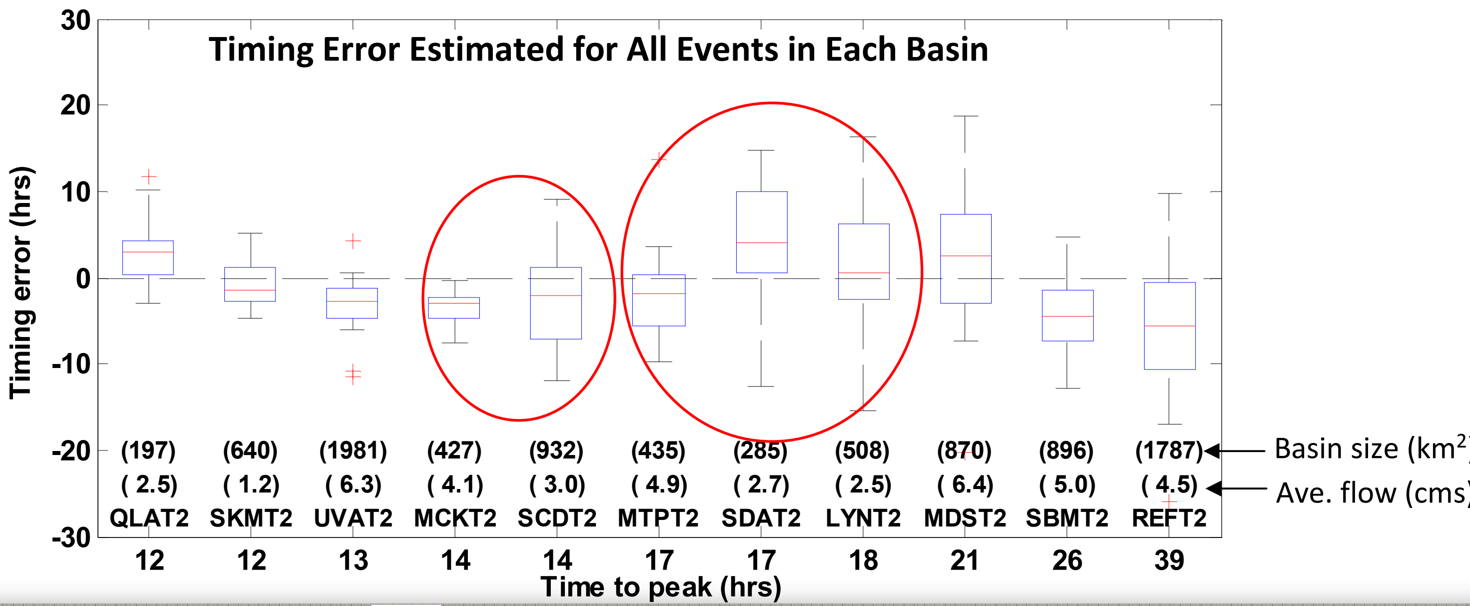


✓1000 MC synthetic simulations generated by applying (to obs) TEs randomly sampled from [-25 +25] hrs.

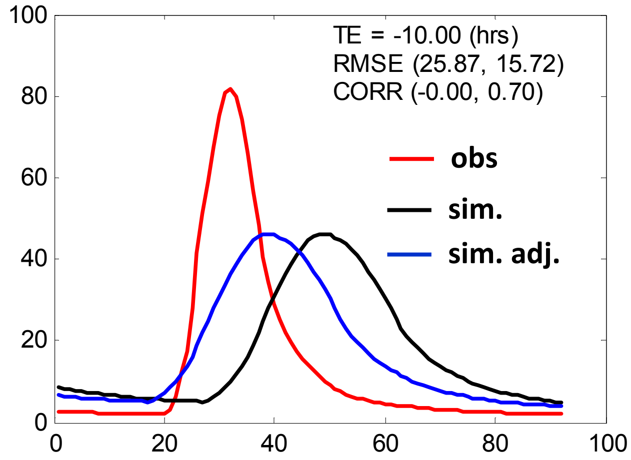
✓ For each simulation, different TE for different events, constant TE within each event

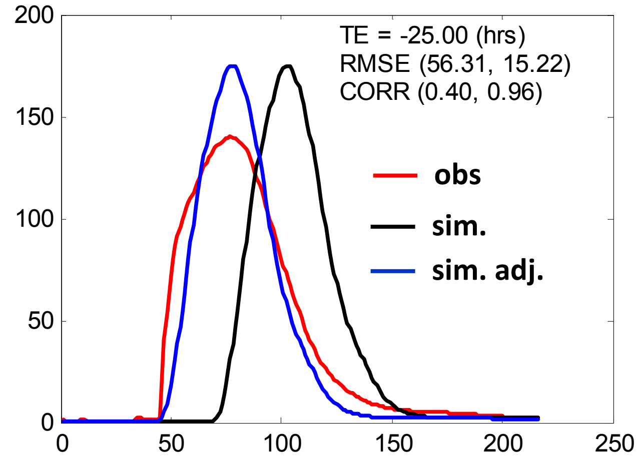
Applying XWT to Real Streamflow Simulations



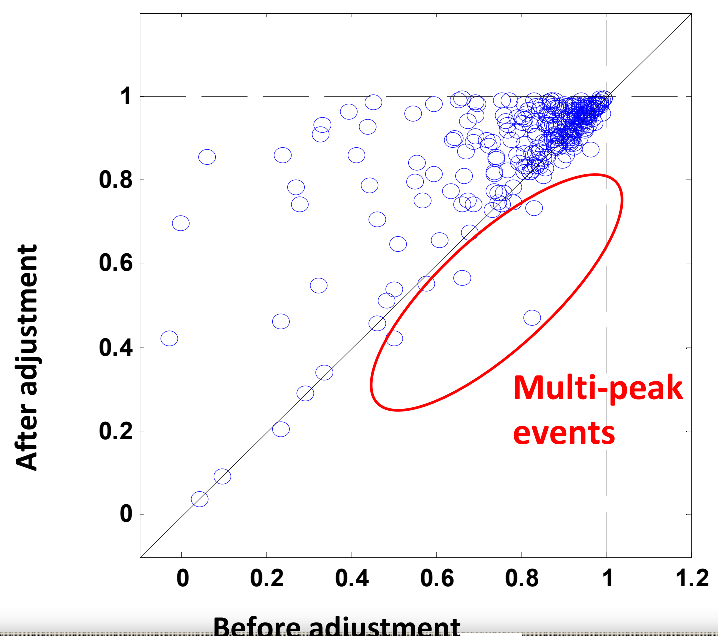


Timing Adjustment Based on Estimated Timing Errors

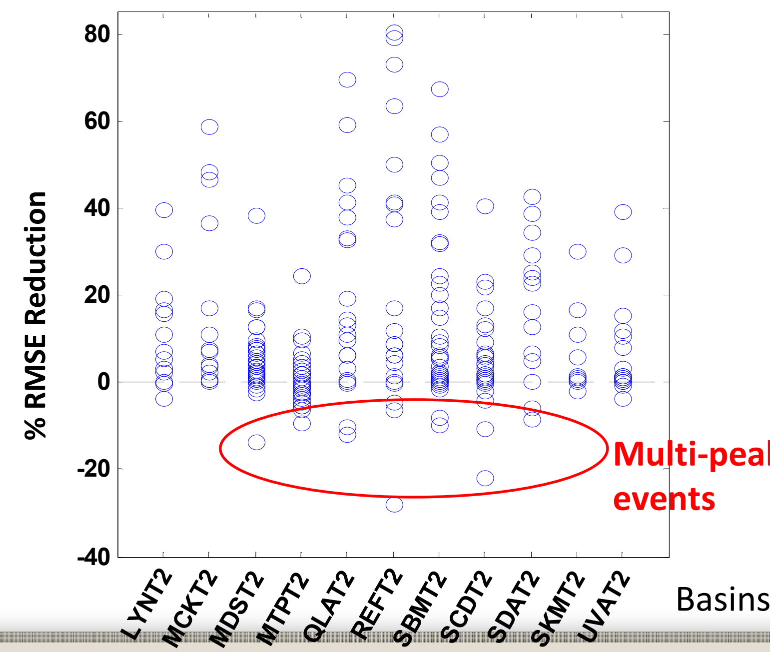




Correlation Coefficient



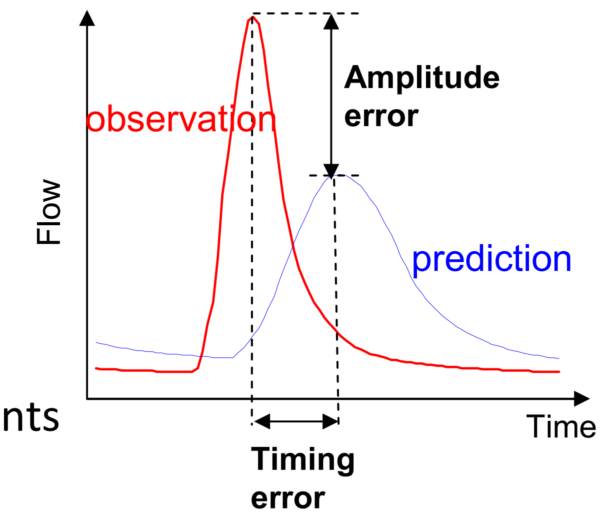
% RMSE Reduction From Time Adjustment



Summary & Future Work

Timing Errors

Different from amplitude errors by nature and should be distinguished in model evaluation and decision support



XWT-based Timing Error Analysis

Holds large potential for quantifying timing errors in streamflow prediction

More reliable for single-peak rather than multi-peak events

Can lead to effective timing adjustment

Future work

Extend to ensemble streamflow prediction

Incorporate into applications of model evaluation & diagnosis, forecast verification, data assimilation, real-time forecasting …