

PROBABILISTIC HYDROLOGIC FORECASTING: AN ENSEMBLE APPROACH

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Abstract: The National Weather Service has been requested by a variety of users to provide hydrologic forecasts that explicitly account for the uncertainty in the forecast. The Ensemble Streamflow Prediction system was constructed to quantify the uncertainty in long range forecasts. It operates using the historical record of precipitation and temperature in combination with the current conditions to produce an ensemble of stream flow time series. This ensemble can be analyzed to produce long range probabilistic forecasts of stream flow, volume, and other predictands. However, in order to account for uncertainty in shorter range forecasts, a probabilistic quantitative precipitation forecast must be used to construct the precipitation time series ingested by the ESP. Furthermore, post processing must be performed on the output ensemble of discharge time series in order to account for biases and errors in the hydrologic models or their calibrations. This paper gives a description of the approach used to construct ensembles for short range probabilistic hydrologic forecasts.

INTRODUCTION

The National Weather Service River Forecast System: The National Weather Service River Forecast System (NWSRFS) is the software used by the River Forecast Centers (RFCs) to produce hydrologic forecasts of river stage, flow, and volume (Fread et al., 1995). It is a deterministic system, meaning that the initial information available to the NWSRFS at forecast time is used to produce only a single forecast of the future hydrologic conditions. To produce a probabilistic forecast, the NWSRFS must be run many times using different input data for each run.

The Ensemble Streamflow Prediction System: The Ensemble Streamflow Prediction system (ESP) is used to construct probabilistic forecasts using the NWSRFS. Described in Perica (1998), the ESP uses historical data in conjunction with the current conditions to produce a forecast. The process is as follows:

1. The NWSRFS is initialized to the conditions at the time the forecast is produced.
2. Each year of the historical record is used as a precipitation and temperature scenario.
3. This scenario is then passed to the NWSRFS as forecasts of future precipitation and temperature.
4. The NWSRFS produces a single future hydrograph, or time series of hydrologic data.
5. This process is repeated until one hydrograph is produced for each year of historical data.

The set, or ensemble, of hydrographs resulting from the ESP can be used to make probabilistic statements about the likelihood of future hydrologic events.

Problem Statement: With the present system, the input used for each run of the NWSRFS is selected using the current conditions in conjunction with historical data and long range forecasts of precipitation and temperature. At no time is the knowledge contained within a short range forecast of future precipitation and temperature included in the process. Such forecasts, which are currently made available on a daily basis to the RFCs with a lead time of up to three days, may contain useful information about the future, information that should be included. However, the current process will not account for such information.

Furthermore, the current process does not account for uncertainty inherent in the models themselves and in the data used to produce the forecasts. Uncertainty in the models may be due to poor calibration. Uncertainty in the data may be due to poor or incomplete measurements. Whatever the source of the uncertainty, the final probabilistic forecast must account for these uncertainties in order to maximize its skill and informativeness.

A New Approach: This paper describes an ensemble approach that accounts for both the uncertainty in the precipitation forecast and in the model. The new approach applies Bayesian logic similar to that described in Krzysztofowicz (1999) to construct ensemble forecasts of precipitation and temperature from a single deterministic forecast. These ensemble forecasts are used in place of the historical data for a run of the ESP. The output from the ESP is then sent through an ensemble post-processor, which modifies it in order to account for model biases and uncertainties. The resulting approach is built around the current components and data streams of the NWSRFS and requires only a one-time calibration.

The following sections describe the ensemble pre-processor and ensemble post-processor, and provides an example to illustrate the approach. For the remainder of this paper, it is assumed that the ESP is being used to produce river discharge hydrographs.

ENSEMBLE PRE-PROCESSOR

Overview: The ensemble pre-processor generates future precipitation and temperature scenarios for use as input to the ESP. The scenarios, when taken as a whole, account for uncertainty about the future precipitation and temperature conditional on the available forecasts of precipitation and temperature. It has a formulation, application, and calibration component, each of which is described below. Only the precipitation ensemble pre-processor is discussed, as the temperature ensemble pre-processor is still under development.

Formulation: Let X be the observed precipitation amount with realization x , and Y be the forecasted precipitation amount with realization y . The goal is to compute the distribution of X given the forecast $Y = y$. The difficulty arises from the unusual nature of precipitation. Its distribution has a discrete component associated with the probability of precipitation and a continuous component associated with the amount that falls in the event that precipitation occurs.

Marginal Distributions of Precipitation Amounts: Let f_X be the density of X and f_Y be the density of Y . In order to account for the probability associated with the observed or forecasted

precipitation amount being zero, density f_X incorporates the Dirac delta function, δ , (Edwards and Penney, 1994) as follows:

$$f_X(x) = (1 - p_{0X})\delta(x) + p_{0X}f_{XC}(x | x > 0),$$

where p_{0X} is the observed probability of precipitation. Hence, the cumulative distribution function, F_X , has the form

$$F_X(x) = 1 - p_{0X} + p_{0X}F_{XC}(x | x > 0).$$

The density f_Y and cdf F_Y have similar forms.

Normal Quantile Transform: The normal quantile transform, or NQT, is a probability mapping of a variate into a standard normal variate (Kelly and Krzysztofowicz, 1997). In this case,

$$z_x = Q^{-1}(F_X(x)),$$

where Q is the standard normal cumulative distribution function. The result is that variate Z_x has the standard normal distribution. This transformation is usually applied to variates whose true distribution can only be estimated empirically, as is the case with precipitation data, in order to acquire normally distributed data that can be more readily analyzed. The inverse normal quantile transform is simply the inverse of the above, or

$$x = F_X^{-1}(Q(z_x)).$$

Applying the inverse NQT to a variate that has been transformed via the NQT will result in the original variate.

Bivariate Distribution of Observed and Forecasted Precipitation: Computation of the bivariate distribution F , with marginals F_X and F_Y , is facilitated by computing a bivariate normal distribution. To begin, variates Z_X and Z_Y are acquired by application of the NQT, described above. Next, the density $\phi(z_X, z_Y)$, with distribution function Φ , is modeled as bivariate normal with standard normal marginals and with parameter ρ , which is the Pearson's correlation coefficient between Z_X and Z_Y . The result of this transformation is that

$$F(x, y) = \Phi(z_X, z_Y; \rho).$$

Characteristics of ρ : The correlation coefficient ρ is dependent on the spatial scale of the forecast, the width of the time interval of the forecast, and the lead time of the forecast. Kelly and Krzysztofowicz (1997) have also shown that ρ is the Spearman's rank correlation coefficient between X and Y in the original space, and serves as a measure of the skill of the forecaster, being 1 for a perfect forecast and 0 for a completely unskilled forecast.

Conditional Distribution of Observed Precipitation: The conditional density $f_C(x | Y = y)$ characterizes the probabilistic forecast of precipitation. Equating F with a bivariate normal

density allows for $f_C(x | Y = y)$ to be computed as the conditional density $\phi_C(z_X | Z_Y = z_Y; \rho)$ which is known to be normal with mean $\mu = \rho z_Y$ and variance $\sigma^2 = (1 - \rho^2)$. This form of the distribution can be viewed as the climatology shifted by the information contained in the forecast, so that as the skill of the forecast decreases (i.e. as ρ goes to 0), the conditional density ϕ_C approaches the density of Z_X , so that $f_C(x | Y = y)$ approaches the marginal density $f_X(x)$.

Application: With the formulation in hand, the next step is determining how to apply this methodology to construct precipitation scenarios. These scenarios, which constitute an ensemble of precipitation time series, will be used as input to the ESP.

Constructing an Ensemble: The ensemble of precipitation amounts is constructed using the climatological record, where each year of data corresponds to one time series in the ensemble. For a given time period and for year k , the process is as follows:

1. The year is ranked according to the amount of precipitation that occurred in that time period during that year relative to other years. For zero events, the ranks are assigned randomly with none to exceed the smallest non-zero event rank.
2. The year has a probability, p_k , assigned to it based upon its rank.
3. The year has a value for variate Z_X assigned to it, which is computed as the inverse of the conditional distribution: $z_{X,k} = \Phi_C^{-1}(p_k | Y = y; \rho)$.
4. The year has a precipitation amount assigned to it by performing the inverse NQT using the marginal of the observed values: $x_k = F_X^{-1}(Q(z_{X,k}))$.

Characteristics of this Ensemble Approach: By using the historical record to construct the ensemble, the spatial and temporal characteristics of the rainfall is captured. For example, if precipitation over two basins is highly correlated, this characteristic will be captured in the climatological record, so that the time series that are constructed will also capture this characteristic. Furthermore, by ranking the zeros and shifting the entire distribution the intermittent character of precipitation is preserved: when more rain falls, it falls on more days and not just in larger amounts.

Calibration: This formulation requires the estimation of conditional distributions F_{XC} and F_{YC} , probabilities p_{0X} and p_{0Y} , and correlation coefficient ρ . Each is computed based upon historical forecasted and observed data, which first passes through the smoothing process described below.

Smoothed Climatology: The distributions of observed and forecasted precipitation amounts are noisy at the daily time step, meaning that the distribution for one day may differ greatly from the distribution on the next day. This is caused by severe storms that are present in the historical record but lasted only a day or two, thus skewing the distribution for those days.

Hence, in order to use the historical data to construct distributions F_{XC} and F_{YC} , the statistics derived from the daily data are first smoothed. Three statistics are the objects of the smoothing for the observed and forecasted data:

- the daily probability of precipitation, or PoP (p_{0X} and p_{0Y})
- the daily average of all non-zero events (the conditional average, or $CAVG_X$ and $CAVG_Y$)
- the daily coefficient of variation of all non-zero events (the conditional coefficient of variation, or CCV_X and CCV_Y)

The smoothing process is two fold. First, the statistic for a particular day and year is computed using the values within a 90 day window centered on that day. Second, the statistic is smoothed with a three component Fourier series. Figure 1 provides an example of smoothed probability of precipitation data.

Calibrated Parameters: Once the above statistics are smoothed, each of the components of the formulation is estimated for a particular day as follows:

- p_{0X} and p_{0Y} : extracted directly from the smoothed statistics.
- F_{XC} and F_{YC} : computed as two parameter distributions estimated using the smoothed $CAVG$ and CCV . Currently, the Weibull or Gamma distribution is used (Evans et al., 1993).
- ρ : computed by using the above estimates to calculate F_X and F_Y , and then calculating Z_X and Z_Y as described above, and computing their correlation coefficient.

The parameters are computed off-line prior to forecast time and the process is fully automated.

ENSEMBLE POST-PROCESSOR

Overview: The ensemble post-processor adjusts the output time series from the ESP in order to account for uncertainty in the hydrologic model contained within the NWSRFS. These uncertainties may be due to errors in the calibration of the model, errors in the initial conditions

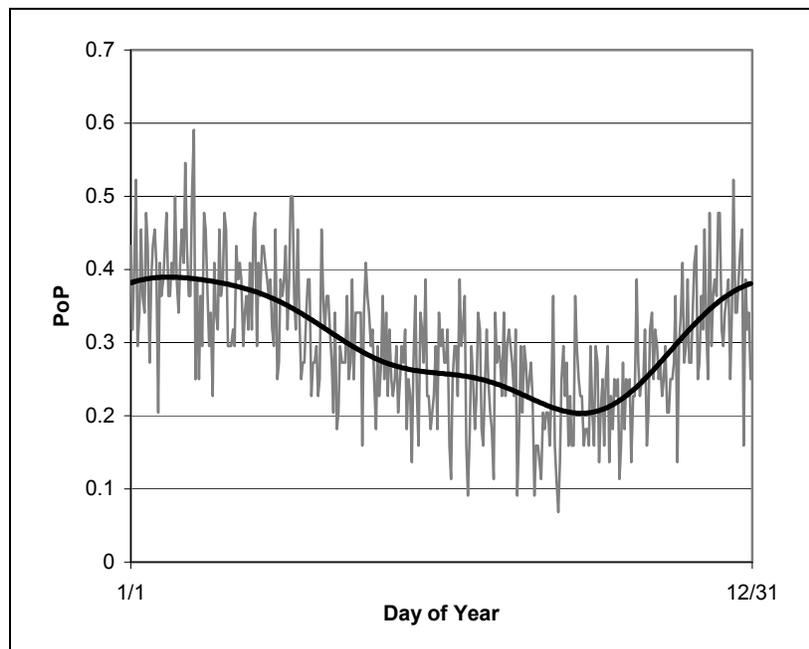


Figure 1: Example of smoothed probability of precipitation data.

of the model, errors in the rating curve, or, in general, any source of error other than future precipitation and temperature. The formulation, application, and calibration components are described below.

Formulation: Let Q_t be the observed discharge at time t with realization q_t and S_t be the simulated discharge at time t under perfectly known future precipitation and temperature with realization s_t . The goal is to compute the distribution of the observed discharge Q_t given the simulated discharge, $S_t = s_t$.

Marginal Distributions of Discharge: Let g_Q be the density of Q_t and g_S be the density of S_t . Both densities have a lower bound and are strictly continuous, having no discrete component. Furthermore, the process is assumed to be stationary within a season so that the density functions are constant for all t within the current season. Next, let $z_{Q,t} = Q^{-1}(G_Q(q_t))$ and $z_{S,t} = Q^{-1}(G_S(s_t))$, so that $Z_{Q,t}$ is the NQT applied to Q_t and $Z_{S,t}$ is the NQT applied to S_t .

Regression: At time t , the information available consists of the simulated value S_t and the previous observed value Q_{t-1} , as well as other earlier values. After applying the NQT to get $Z_{S,t}$ and $Z_{Q,t-1}$, the following auto-regression model with lag 1 is used in the Gaussian space to estimate $Z_{Q,t}$:

$$z_{Q,t} = az_{Q,t-1} + bz_{S,t} + \varepsilon; \quad \varepsilon \sim N(0, \sigma_\varepsilon^2)$$

Once again, under the assumption that the process is stationary within a season, coefficients a and b and residual one-step variance σ_ε^2 are constant for all t within the current season.

Conditional Distribution of Observed Discharge: By the nature of the NQT, it is true that

$$P(Q_t \leq q_t \mid S_t, Q_{t-1}) = P(Z_{Q,t} \leq z_{Q,t} \mid Z_{S,t}, Z_{Q,t-1})$$

and, using the regression above,

$$(Z_{Q,t} \mid Z_{S,t}, Z_{Q,t-1}) \sim N(az_{Q,t-1} + bz_{S,t}, \sigma_\varepsilon^2).$$

These equations allow us to compute the distribution of discharge at time t given the simulated value at time t and the observed value at time $t - 1$.

Application: It remains to determine how to apply the formulation to the ensemble of hydrographs, or time series, produced by the ESP. Let one such time series be denoted by q_t , $t = 1, \dots, n$. There are two possible application models: a deterministic model and a stochastic model. Both are applied independently to each times series within the ensemble of hydrographs, with each application resulting in an adjusted times series, q_t^* , $t = 1, \dots, n$.

Deterministic Model: The deterministic model consists of computing the expected value of the observed discharge conditional on the expected value of the observed discharge in Gaussian space, as calculated using the formulation above. The model is applied recursively as follows:

1. For time $t = 1$, compute the expected value

$$\underline{z}_{Q,1} = E[Z_{Q,1} | Z_{S,1}, Z_{Q,0}] = az_{S,1} + bz_{Q,0} ,$$

where $z_{Q,0}$ is the NQT transformed observed value at the time the forecast is produced.

2. For times $t = k$, $k = 2, \dots, n$, compute the expected value

$$\underline{z}_{Q,k} = E[Z_{Q,k} | Z_{S,k}, Z_{Q,k-1} = \underline{z}_{Q,k-1}] = az_{S,k} + bz_{Q,k-1} ,$$

where $\underline{z}_{Q,k-1}$ is the expected value computed at time previous time step, $k - 1$.

3. For all times t , $t = 1, \dots, n$, the adjusted time series value is computed as the expected value of the observed discharge conditional on the simulated and previous observed values in Gaussian space, or

$$q_t^* = E[Q_t | Z_{S,k}, Z_{Q,k-1}] .$$

This computation makes use of the expected value computed in step 2, but is not closed form, requiring numerical integration to complete.

Stochastic Model: The stochastic model is similar to the deterministic model, except that the adjusted time series value is the inverse NQT applied to a single random sample drawn from the distribution of the observed value in Gaussian space. Hence, the stochastic model follows the same first two steps as the deterministic version, with step 3 being as follows:

3. For all times t , $t = 1, \dots, n$, compute a random realization, $z_{r,t}$, of the variate $(Z_{Q,t} | Z_{S,t}, Z_{Q,t-1}) \sim N(\underline{z}_{Q,t}, \sigma_\varepsilon^2)$, and apply the inverse NQT to this realization to acquire the adjusted time series value: $q_t^* = G_Q^{-1}(Q(z_{r,t}))$.

Implications: For the stochastic model, the recursive nature of the application leads to the following:

$$(Z_{Q,k} | Z_{S,k}, Z_{Q,k-1}) \sim N\left(a^k z_{Q,0} + b \sum_{i=0}^{k-1} a^i z_{S,k-i}, \sum_{j=0}^{k-1} a^{2(i-1)} \sigma_\varepsilon^2\right)$$

Thus, the forecast distribution at time $t = k$, $k > 0$, is a weighted sum of the observed value at the time the forecast is produced, the simulated value at time $t = k$, and all of the simulated values preceding time $t = k$. Note that, under the constraint that $0 < a < 1$, as the lead time of the forecast increases, the weight on the observed at the time the forecast is produced decreases, as does that for the earlier simulated values.

Calibration: Calibration of the ensemble post-processor requires estimation of the marginal distributions G_Q and G_S , and regression parameters a , b , and σ_ε^2 . The estimation is performed using historical records of the observed discharges and corresponding simulated discharges. Such a historical simulation can be produced via ESP.

Estimating G_Q and G_S : G_Q and G_S are estimated empirically. Simply collect all of the observed and simulated data to be used in the calibration, sort them, rank them, and assign probabilities accordingly.

Estimating a , b , and σ_ϵ^2 : As opposed to a typical linear regression, parameters a and b are estimated by minimizing the errors relative to Q_t , not relative to $Z_{Q,t}$. Hence, an optimization must be performed in which the variables that are being optimized are a and b , and the objective is to minimize an error measure between samples of Q_t and the inverse NQT transformed values of $z_{Q,t}$ computed using the regression equation. Regardless of how a and b are estimated, the residual variance, σ_ϵ^2 , is always computed using the values in the Gaussian space.

Seasonal and High Flow Dependency: The behavior of a river may be dependent on the time of the year. As such, it may be desirable to derive seasonal estimates of the parameters, so that the parameters used to apply the ensemble post-processor depend on the day the forecast is being produced.

Furthermore, the quality of the model often depends on the discharge level. Hence, it may be desirable to derive the regression parameters separately for high and low discharges. However, the empirical estimates of G_Q and G_S should be the same for both.

EXAMPLE

Overview: The example provided below illustrates both the ensemble pre-processor and post-processor. It is a real-life example, generated at the Mid-Atlantic River Forecast Center on April 24, 2002, for the station in Huntingdon, PA.

Description: The details of the calibration and application of the ensemble pre-processor and ensemble post-processor are provided below.

Ensemble Pre-Processor Calibration: The parameters for the ensemble pre-processor were calibrated at 6 hour time steps using a record consisting of two years of observed and forecasted precipitation data. The forecasts are those generated for the RFCs via the standard NWS quantitative precipitation forecast process. Table 1 provides the statistics $CAVG_X$, $CAVG_Y$, CCV_X , and CCV_Y , the values of p_{0X} and p_{0Y} , and the correlation coefficient ρ . Figure 2 is a plot of the distributions F_X and F_Y .

Ensemble Pre-Processor Application: The forecast of 6 hour precipitation amounts for the period from April 24 at 12 UTC to April 26 at 12 UTC was all zeros except for the time period from 6 UTC to 12 UTC on April 25, which had a value of 0.26 in. For that 6 hour time period,

	PoP	CAVG	CCV	ρ
X (Obs.)	0.216	0.172	1.825	----
Y (Fcst.)	0.313	0.116	1.015	0.853

Table 1: Parameters of ensemble pre-processor.

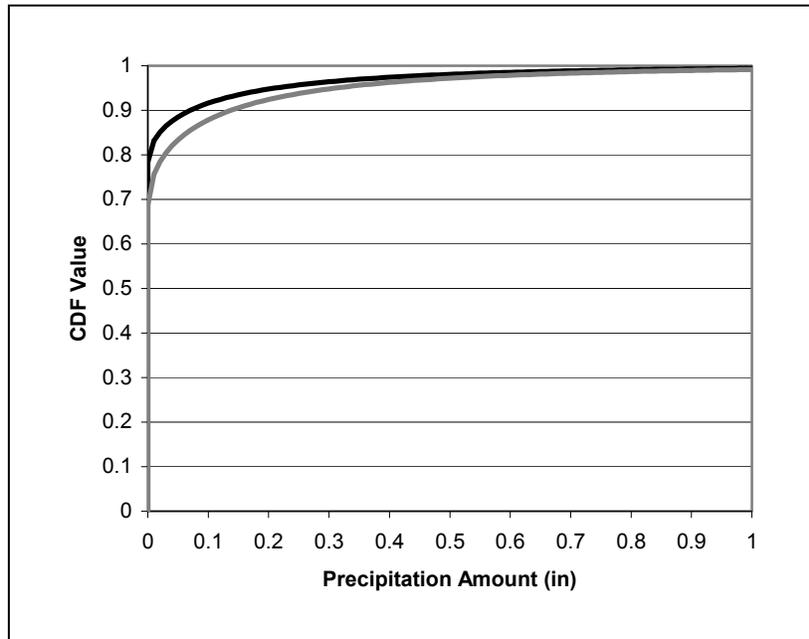


Figure 2: Marginal distribution F_X (black) and F_Y (gray) used in ensemble pre-processor.

Figure 3 provides plots of the historical, unsmoothed distribution of observed precipitation and the distribution of precipitation amounts resulting from the ensemble pre-processor. Both are empirical estimates.

Ensemble Post-Processor Calibration: The parameters for the ensemble post-processor were calibrated using a record consisting of 37 years of observed and simulated mean daily discharge. Calibration was performed for the season consisting of April and May, and the discharge values were broken down based on those below 400 CMS and above 400 CMS. Table 2 provides the parameters a , b , and σ_ϵ^2 for both above and below 400 CMS. Figure 4 provides the empirical estimates of distributions G_Q and G_S .

Ensemble Post-Processor Application: Figure 5 provides the ensemble of hydrographs that was produced by ESP, the adjusted ensemble produced by application of the deterministic model, and the ensemble produced by application of the stochastic model.

Discussion: Two observations that can be made pertaining to this example will now be discussed. First, from Figures 2 and 3, it is seen that, at the six hour time scale, the probability

	a	b	σ_ϵ^2
Below	0.734	0.281	0.042
Above	0.104	0.883	0.076

Table 2: Parameters of ensemble post-processor.

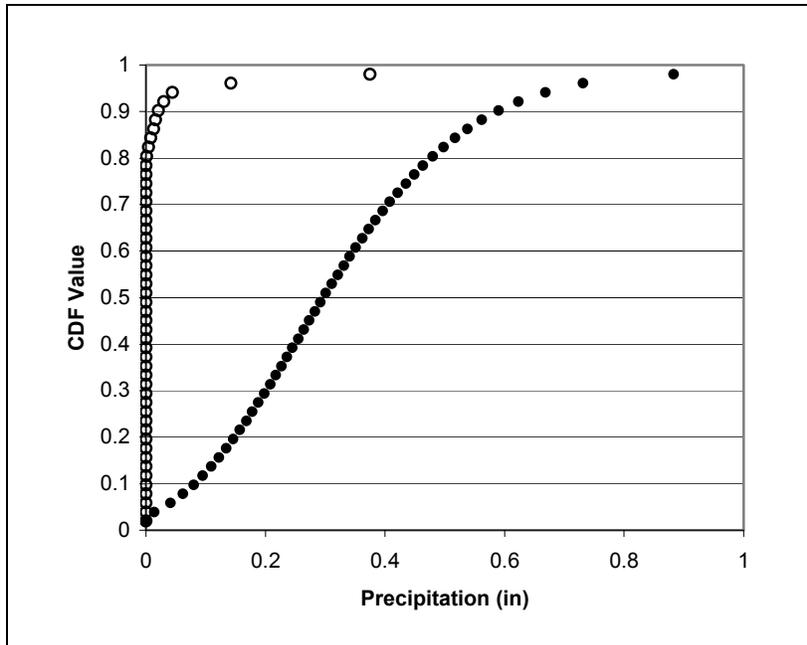


Figure 3: Empirical distribution of original precipitation data for hours 6 – 12 UTC on April 25 (hollow circles) and distribution resulting from application of ensemble pre-processor (black dots).

of precipitation is typically less than 30%. Thus, the impact of how the zero precipitation events are ranked, which is currently randomly, is strongly felt. So a more intelligent “nearest neighbor” ranking technique is needed, in which the zero events are ranked based on proximity to non-zero events both in time and in space. Such a technique is being researched at this time.

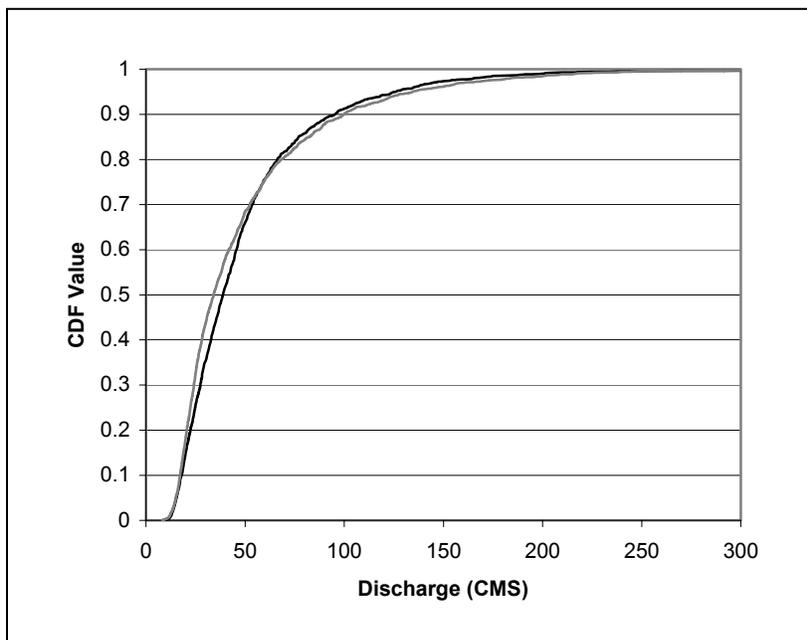


Figure 4: Empirical estimates of marginal discharge distributions G_Q and G_ξ .

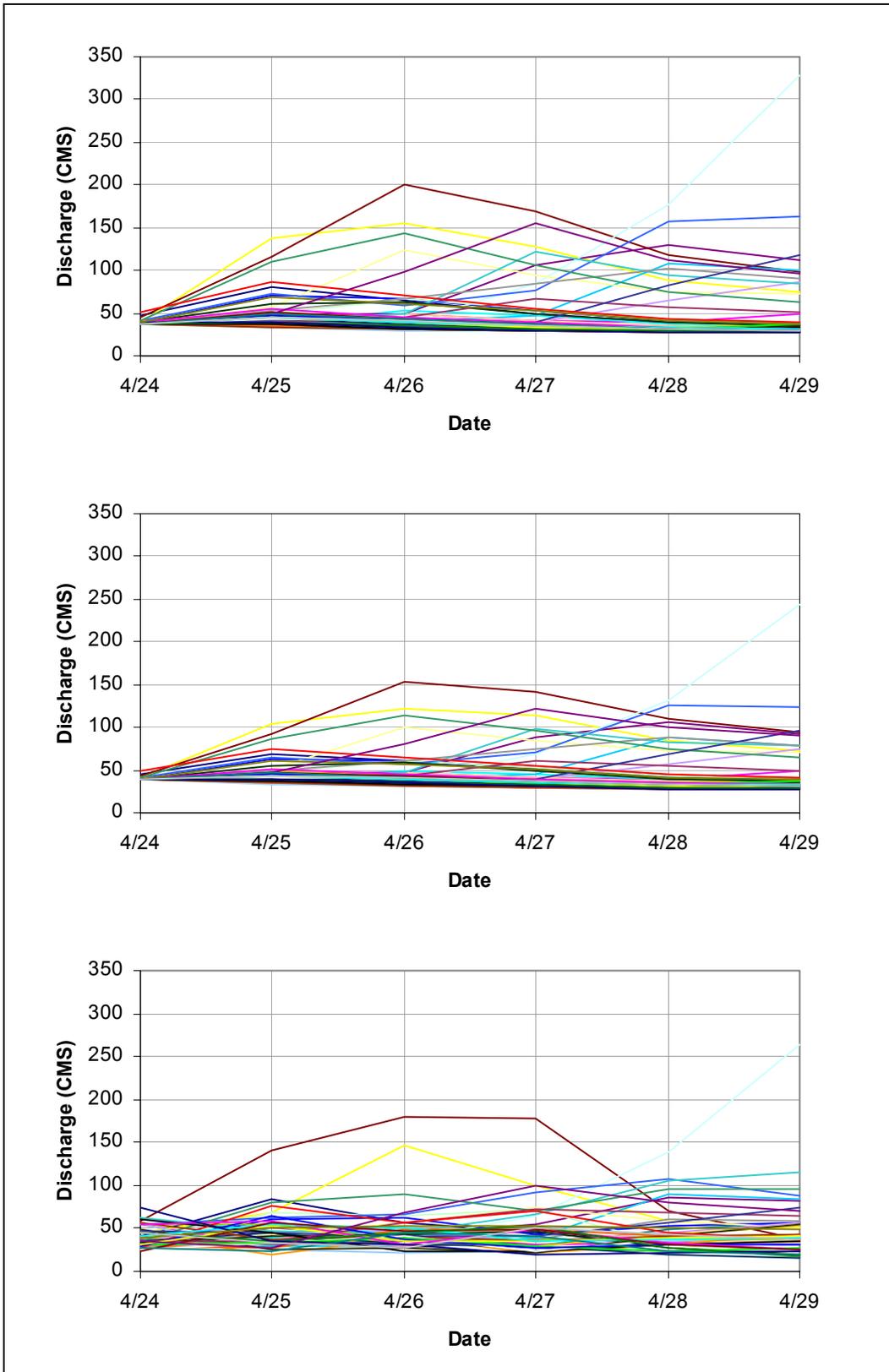


Figure 5: The original ESP output hydrographs (top), the deterministic model adjusted hydrographs (middle), and the stochastic model adjusted hydrographs (bottom).

Second, both the deterministic and stochastic models of the ensemble post-processor adjust the output time series from the ESP. This adjustment is calculated in such a way that a time series adjusted by the ensemble post-processor is useless when considered individually, as it will not represent an actual feasible event. This fact is visible in Figure 5 for the stochastic model. Hence, you must take all of the time series as a whole in order to understand the statistical properties of the future event.

In general, the real life example above illustrates how the methodology described herein is applied to generate an ensemble of precipitation amounts that accounts for meteorological and model uncertainty.

CONCLUSION

At this time, the NWS is advancing the hydrologic science used to forecast rivers by enhancing the ensemble forecast process. Substantial work has been done to develop pre-processing and post-processing algorithms. The algorithms developed use existing NWS data streams and have proven successful over the Juniata river basin in Pennsylvania. The pre-processor captures meteorological uncertainty, or uncertainty about future precipitation and temperature, perhaps the most significant source of uncertainty in forecasting hydrological events. As meteorology improves, the pre-processing algorithm will incorporate new forecasts with longer lead times. In this way, the hydrologic forecasts will be able to take full advantage of any advancement in the skill of meteorological forecasts. The post-processor captures the non-meteorological uncertainty, or model uncertainty, which is a major source of uncertainty for short term forecasts. It is these short term forecasts that end users are demanding. Further refinements of both algorithms will be directed by the results of the developing implementation efforts.

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