EXPLICIT DIFFUSION IN GFDL FV^3

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1. Divergence damping

Horizontal divergence (along a Lagrangian surface) is computed as a cell-integrated quantity on the dual grid:

$$D = \frac{1}{\Delta A_c}\left[\delta_x\left(u_c \Delta y_c \sin\alpha\right) + \delta_y\left(v_c \Delta x_c \sin\alpha\right)\right] \tag{1}$$

(A complete list of variables is given at the end of the document.) The Laplacian of D can also be computed as a cell-integrated quantity on the dual grid:

$$\nabla^2 D = \frac{1}{\Delta A_c}\left[\delta_x\left(\frac{\delta_x D}{\Delta x}\Delta y_c \sin\alpha\right) + \delta_y\left(\frac{\delta_y D}{\Delta y}\Delta x_c \sin\alpha\right)\right] \tag{2}$$

This operator can be applied on $\nabla^2 D$ instead of D to yield $\nabla^4 D$. The damping is then applied when the forward timestep is taken for the horizontal dynamics along vertically-Lagrangian surfaces:

$$u^{n+1} = u^n + \cdots + \nu_D \frac{\delta_x \nabla^{2N} D}{\Delta x} \tag{3}$$

$$v^{n+1} = v^n + \cdots + \nu_D \frac{\delta_y \nabla^{2N} D}{\Delta y} \tag{4}$$

where N (equal to the namelist parameter `nord`) is 1 for fourth-order and 2 for sixth-order damping. The nondimensional damping coefficient is given as

$$\nu_D = \left(d_4 \Delta A_{\min}\right)^{N+1} \tag{5}$$

in which d_4 is the parameter `d4_bg` in the namelist, and $\Delta A_{\min}$ is the *global* minimum grid-cell area. It is recommended that this parameter be set to a value between 0.1 and 0.16, with instability likely for higher or lower values. Note that divergence damping is necessary as there is no implicit damping on the divergence in FV^3.

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2. Vorticity damping

Vertical vorticity is a cell-integrated quantity on the model grid. The vorticity fluxes $v\zeta/\Delta x$ and $-u\zeta/\Delta y$ are used to update the vector-invariant momentum equations. We can apply damping on the vorticity as well; to maintain consistent advection, the same damping is applied to the mass, heat, and vertical momentum fields, all of which are co-located with the vorticity. This additional damping is beneficial when using a non-monotonic advection scheme, which lacks the implicit diffusion of monotonic advection.

The diffusion computed for each variable is computed as diffusive fluxes. Second-order damped vorticity is computed as:

$$f_{x2} = \frac{\delta_x \zeta}{\Delta x_c} \sin \alpha \Delta y \tag{6}$$

$$f_{y2} = \frac{\delta_y \zeta}{\Delta y_c} \sin \alpha \Delta x \tag{7}$$

Again, higher order damping can be achieved by repeatedly applying the Laplacian operator. To get a fourth-order damping (f_{x4} and f_{y4}), apply the above operation on the diffused vorticity:

$$\zeta_2 = -\frac{1}{\Delta A} \left[\delta_x f_{x2} + \delta_y f_{y2}\right] \tag{8}$$

Sixth-order damping can be achieved by performing another iteration to find ζ_4 and f_{x6}, f_{y6}. The forward timestep for the momentum is then evaluated as:

$$u^{n+1} = u^n + \cdots + \nu_\zeta \frac{f_{y(2N)}}{\Delta x} \tag{9}$$

$$v^{n+1} = v^n + \cdots - \nu_\zeta \frac{f_{x(2N)}}{\Delta y} \tag{10}$$

where the nondimensional damping coefficient ν_ζ is evaluated the same way as ν_D, but in which the namelist parameter `vtdm4` is used for d_4. This damping coefficient should be much smaller than that for divergence, since even in a non-monotonic advection scheme, the upstream numerics still has some implicit diffusion, whereas divergence is not explicitly advected.

3. Energy-, momentum-, and mass-conserving $2\Delta z$ filter

FV^3 has the option to use a local, vertical mixing to help remove dynamic instabilities during the simulation. This is similar to the Richardson-number based subgrid-scale diffusion formulations of Lilly (1962, Tellus) and of Smagorinsky (1963), although

their isotropic formulations have been simplified so as to only act on vertical gradients and perform diffusion in the vertical. This diffusion is completely local ($2\Delta z$), diagnosing and acting only on adjacent grid cells.

We compute the local Richardson number on cell interfaces; recall in FV^3 that $k = 1$ is the top layer of the domain and the index increases downward:

$$\mathrm{Ri}_{\mathrm{k}-\frac{1}{2}} = \frac{g\delta z\, \delta_z \theta_v}{(\theta_v^k + \theta_v^{k-1})((\delta_z u)^2 + (\delta_z v)^2)} \tag{11}$$

If $\mathrm{Ri} < 1$, then mixing is performed. The amount of mass transfer between the layers if there is complete mixing is:

$$M_0 = \frac{\delta p^k \delta p^{k-1}}{\delta p^k + \delta p^{k-1}} \tag{12}$$

Here, we scale the amount of mixing so that the mass transferred is $M = M_0 \left(1 - \mathrm{Ri}\right)^2$. Complete mixing is performed when $\mathrm{Ri} \leq 0$. A timescale τ, equal to the parameter `fv_sg_adj`, for this mixing is applied so that the rate at which the mixing occurs can be controlled at run time. For any conserved variable ϕ (including tracer mass, momentum, or total energy) the time tendency produced by the mixing is:

$$\frac{\partial \phi^k}{\partial t} = -\frac{M}{\delta p^k}\left(\phi^k - \phi^{k-1}\right)\frac{1}{\tau} \tag{13}$$

$$\frac{\partial \phi^{k-1}}{\partial t} = +\frac{M}{\delta p^k}\left(\phi^k - \phi^{k-1}\right)\frac{1}{\tau} \tag{14}$$

Note that since total energy and momentum are both conserved, lost kinetic energy automatically becomes heat.

The timescale τ must be larger than the physics timestep (`dt_atmos` in the namelist) to avoid suppressing resolved convective motions. In the $\frac{1}{8}^\circ$ NGGPS idealized tropical cyclone test, $\tau = 1800$ s, compared to a physics timestep of 75 s. In $\frac{1}{4}^\circ$ HiRAM seasonal predictions $\tau = 3600$, compared to the physics timestep of 600 s.

This mixing can be either applied throughout the domain, or only near the model top, to remove instabilities caused by vertically-propagating waves near the top of the domain. The namelist variable `n_sponge` controls the number of levels at the top of the domain to which the filter is applied; if it is equal to `npz` the filter is applied everywhere.

4. Variables and notation

u, v	D-grid winds
u_c, v_c	C-grid winds, at the $t^{n+\frac{1}{2}}$ timelevel.
δp	Layer hydrostatic pressure thickness, proportional to mass
δz	Layer geometric depth for nonhydrostatic solver
θ_v	Virtual potential temperature
ΔA, Δx, Δy	D-grid cell areas and cell face lengths
ΔA_c, Δx_c, Δy_c	dual-grid cell areas and cell face lengths
α	local angle between coordinate axes; $\frac{\pi}{2}$ for an orthogonal coordinate
k	vertical index
n	time index

The differencing notation used in this document follows that of Lin and Rood (1996, 1997) and of Lin (2004), in which the operator $\delta_x \phi$ is defined as a centered-difference operator:

$$\delta_x \phi_{i+1/2} = \phi_{i+1} - \phi_i. \tag{15}$$

The indices on dependent variables are suppressed unless explicitly needed. Note that this differs from the equivalent operator of Durran (1999, 2010) in that it lacks the $\frac{1}{\Delta x}$ term needed to complete the discrete derivative.