FV^3

The GFDL Finite-Volume Cubed-sphere Dynamical Core

Lucas Harris and Shian-Jiann Lin
NOAA/Geophysical Fluid Dynamics Laboratory

27 January 2016

FV^3

- Scalable, flexible dynamical core capable of both hydrostatic and nonhydrostatic simulation

- Successor to latitude-longitude FV core in NASA GEOS, GFDL AM2.1, and CAM-FV

- GFDL models
  - AM4/CM4
  - HiRAM
  - CM2.5/2.6/3
- CAM-FV^3
- LASG
- GEOS-CHEM (coming soon!)
- GISS ModelE

FV^3 Design Philosophy

- Discretization should be guided by physical principles as much as possible
  - Finite-volume, integrated form of conservation laws
  - Upstream-biased fluxes
- Operators “reverse engineered” to achieve desired properties
- Computational efficiency is **crucial**. A fast model is a good model!
  - Solver should be built with vectorization and parallelism in mind
- Dynamics isn’t the whole story! Coupling to physics and the ocean is **important.**

Development of the FV^3 core

- Lin and Rood (1996, MWR): Flux-form advection scheme
- Lin and Rood (1997, QJ): FV shallow-water solver
- Lin (1997, QJ): FV Pressure Gradient Force
- Lin (2004, MWR): Vertically-Lagrangian discretization
- Putman and Lin (2007, JCP): Cubed-sphere advection
- Harris and Lin (2013, MWR): Describes FV^3 and grid nesting
- Lin (in prep): Nonhydrostatic dynamics

Development of the FV^3 core

- **Lin and Rood (1996, MWR): Flux-form advection scheme**
- Lin and Rood (1997, QJ): FV shallow-water solver
- Lin (1997, QJ): FV Pressure Gradient Force
- Lin (2004, MWR): Vertically-Lagrangian discretization
- Putman and Lin (2007, JCP): Cubed-sphere advection
- Harris and Lin (2013, MWR): Describes FV^3 and grid nesting
- Lin (in prep): Nonhydrostatic dynamics

Lin and Rood (1996, MWR)
Flux-form advection scheme

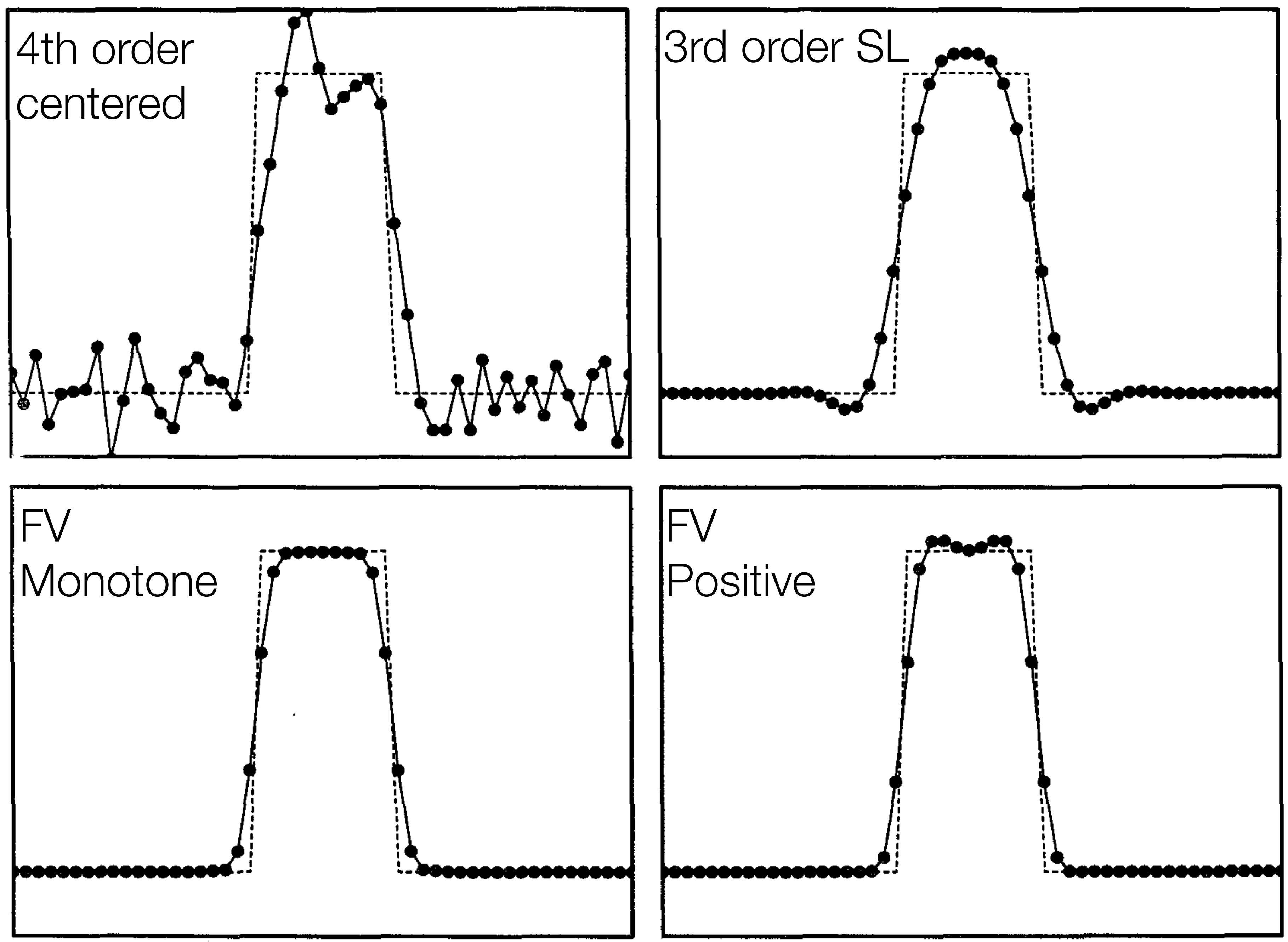
$$q^{n+1} = \frac{1}{\pi^{n+1}} \left\{ \pi^n q^n + F\left[q^n + \frac{1}{2} g(q^n) \right] + G\left[q^n + \frac{1}{2} f(q^n) \right] \right\}.$$

- 2D scheme derived from 1D PPM operators
- Advective form inner operators f, g, allow elimination of leading-order deformation error
    - Allows preservation of constant tracer field under nondivergent flow
- Flux-form outer operators F, G ensure mass conservation

Lin and Rood (1996, MWR) Flux-form advection scheme

- PPM operators are upwind biased
  - More physical, but also more diffusive
- Monotonicity/positivity constraint: important (implicit) source of model diffusion and noise control
  - Also available: linear advection schemes with a selective filter to suppress $2\Delta x$ noise. These can be useful in very high-res nonhydrostatic runs
- Scheme maintains linear correlations between tracers when unlimited or when monotonicity constraint applied (not necessarily so for positivity)

1D Advection Test



Lin and Rood 1996, MWR

Development of the FV^3 core

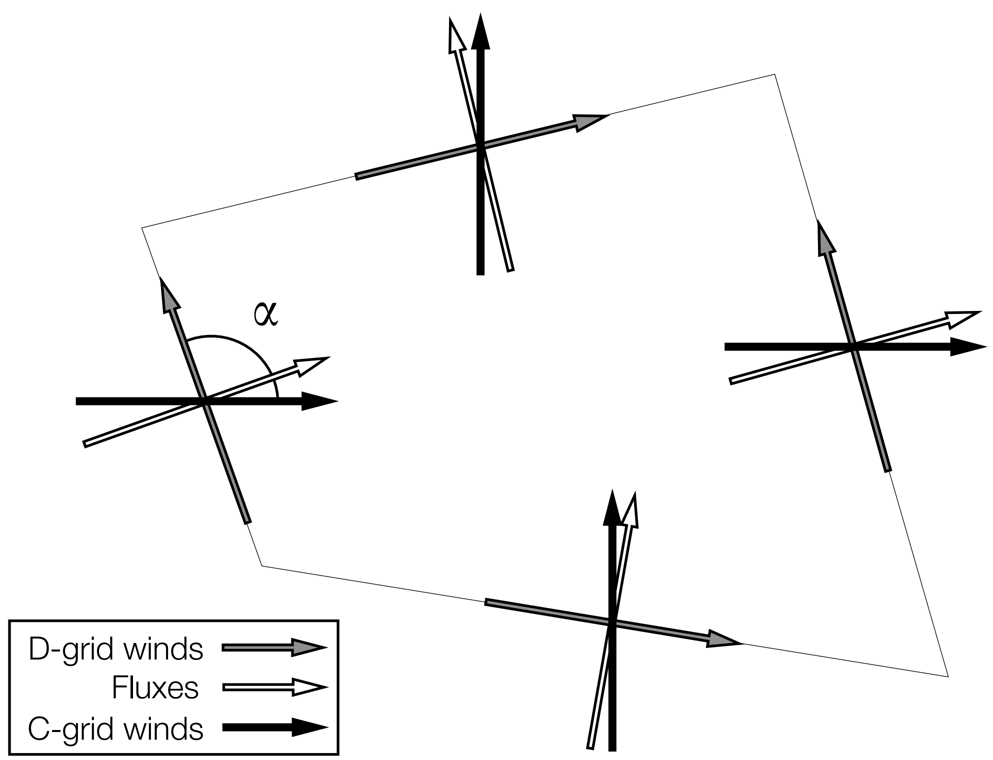
- Lin and Rood (1996, MWR): Flux-form advection scheme
- **Lin and Rood (1997, QJ): FV shallow-water solver**
- Lin (1997, QJ): FV Pressure Gradient Force
- Lin (2004, MWR): Vertically-Lagrangian discretization
- Putman and Lin (2007, JCP): Cubed-sphere advection
- Harris and Lin (2013, MWR): Describes FV^3 and grid nesting
- Lin (in prep): Nonhydrostatic dynamics

Lin and Rood (1997, QJ)
FV shallow-water solver

- Solves layer-averaged vector-invariant equations
- δp is proportional to layer mass
- θ: not in SW solver but is in full 3D Solver
- Forward-backward timestepping
    - PGF evaluated backward with updated pressure and height

$$
\begin{aligned}
\frac{\partial \delta p}{\partial t} + \nabla \cdot (\mathbf{V} \delta p) &= 0 \\
\frac{\partial \delta p \Theta}{\partial t} + \nabla \cdot (\mathbf{V} \delta p \Theta) &= 0
\end{aligned}
$$

$$
\frac{\partial \mathbf{V}}{\partial t} = -\Omega \hat{k} \times \mathbf{V} - \nabla \left(\kappa + \nu \nabla^2 D \right) - \frac{1}{\rho} \nabla p \Big|_z
$$



Lin and Rood (1997, QJ)
FV shallow-water solver

- Discretization on D-grid, with C-grid winds used to compute fluxes

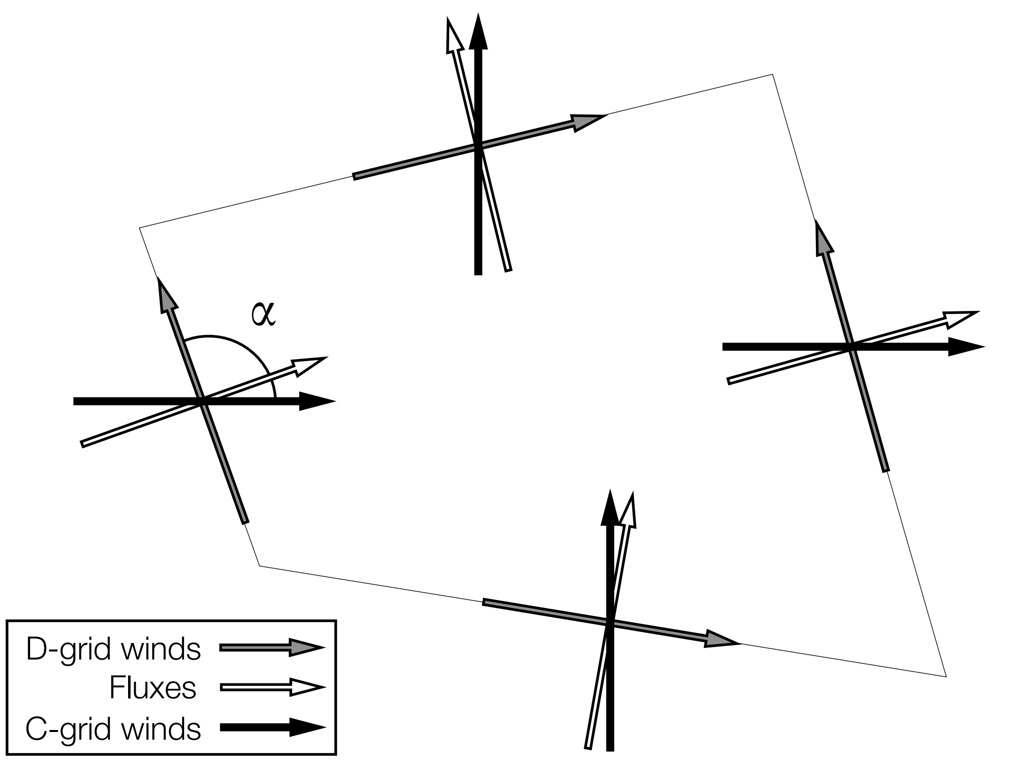
- D-grid winds interpolated to get C-grid winds, which are stepped forward a half-step for an approx. to time-centered winds—a simplified Riemann solver

  - Advantages of D-grid are preserved, and diffusion due to C-grid averaging is alleviated

- Two-grid discretization and time-centered fluxes avoid computational modes

$$\begin{aligned}\frac{\partial \delta p}{\partial t} + \nabla \cdot (\mathbf{V}\delta p) &= 0 \\ \frac{\partial \delta p \Theta}{\partial t} + \nabla \cdot (\mathbf{V}\delta p \Theta) &= 0\end{aligned}$$

$$\frac{\partial \mathbf{V}}{\partial t} = -\Omega \hat{k} \times \mathbf{V} - \nabla\left(\kappa + \nu \nabla^2 D\right) - \frac{1}{\rho}\nabla p\Big|_z$$

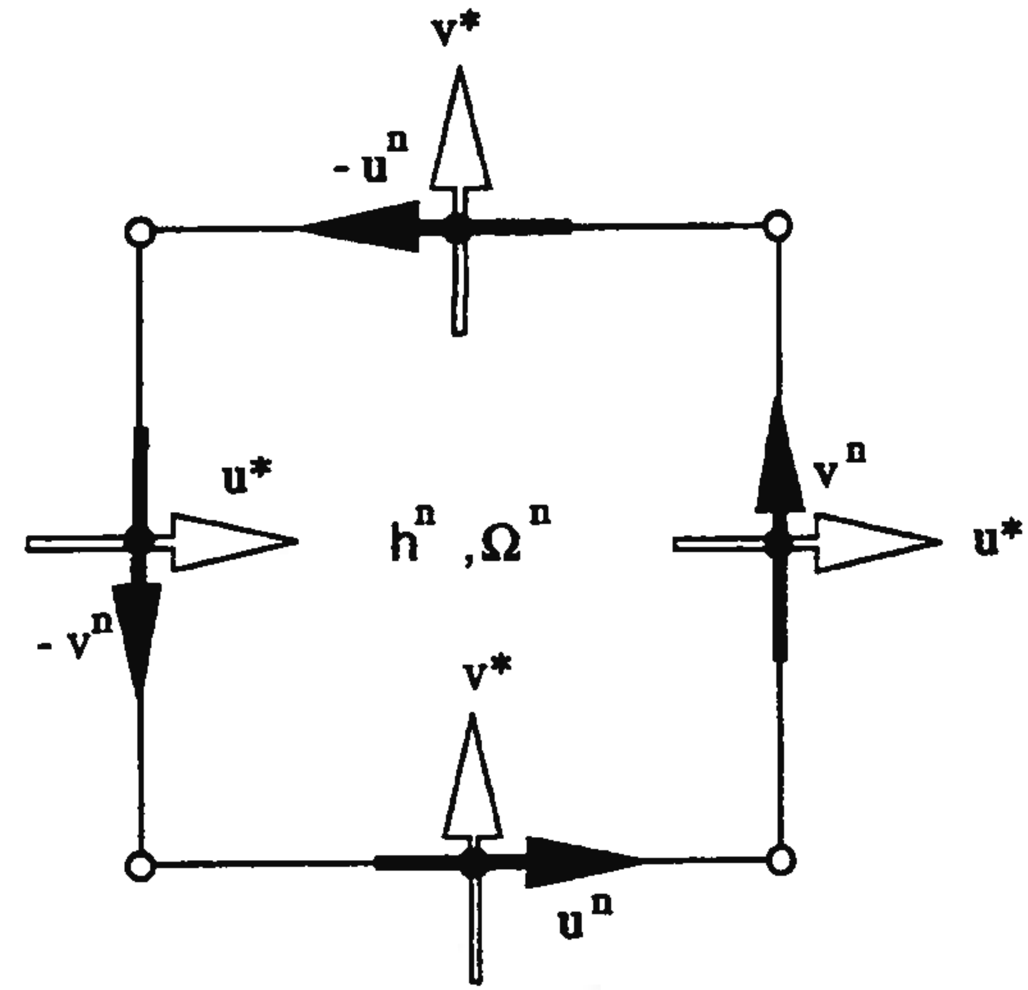


FV shallow-water solver: Time-stepping procedure

- Interpolate time t^n D-grid winds to C-grid
- Advance C-grid winds by one-half timestep to time $t^{n+1/2}$
  - Used as approximation to time-averaged winds for time-averaged fluxes
- Use time-averaged air mass fluxes to update δp and θ to time t^{n+1}
- Compute vorticity flux and KE gradient to update D-grid winds to time t^{n+1}
- Use time t^{n+1} δp and θ to compute PGF to complete D-grid wind update

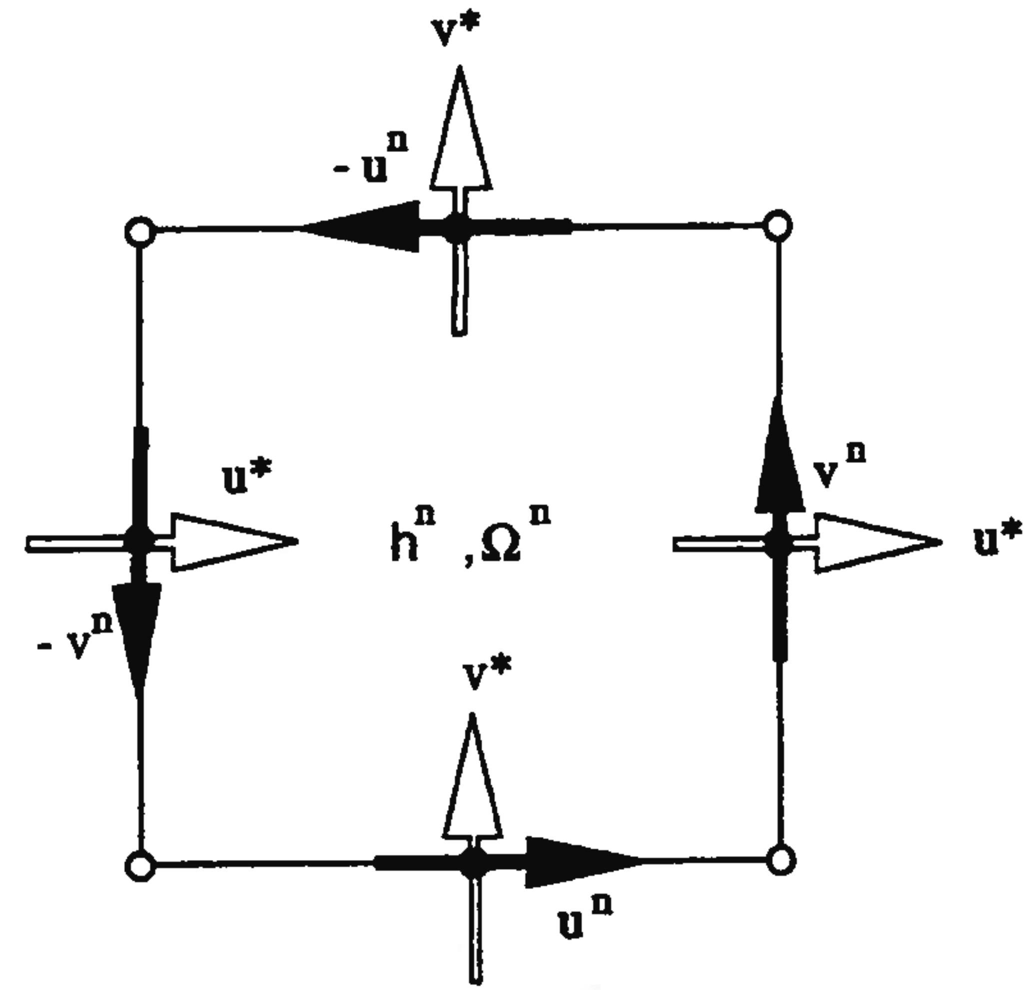
FV shallow-water solver: Vorticity flux

- Nonlinear vorticity flux term in momentum equation
- D-grid allows exact computation of absolute vorticity—no averaging!
- Advantages to this form not apparent in linear analyses



FV shallow-water solver: Vorticity flux

- Vorticity uses same flux as δp
  - Consistent flux of mass and vorticity improves preservation of geostrophic balance.
- Consistent flux also means PV is advected as a scalar!
  - PV is thus **conserved** in adiabatic shallow-water flow.



FV shallow-water solver: Kinetic Energy Gradient

- Vector-invariant equations susceptible to Hollingsworth-Kallberg instability if KE gradient not consistent with vorticity flux

- Solution: use C-grid fluxes again to advect wind components, yielding an upstream-biased kinetic energy

$$\kappa^* = \frac{1}{2}\left\{\mathscr{X}(\overline{u^*}^{\theta}, \Delta t; u^n) + \mathscr{Y}(\overline{v^*}^{\lambda}, \Delta t; v^n)\right\}.$$

    - Consistent advection again!

FV shallow-water: Polar vortex test

- Note how well strong PV gradients are maintained
- Vorticity isn't just important for large-scale flow. Many mesoscale flows are also governed by vorticity too!

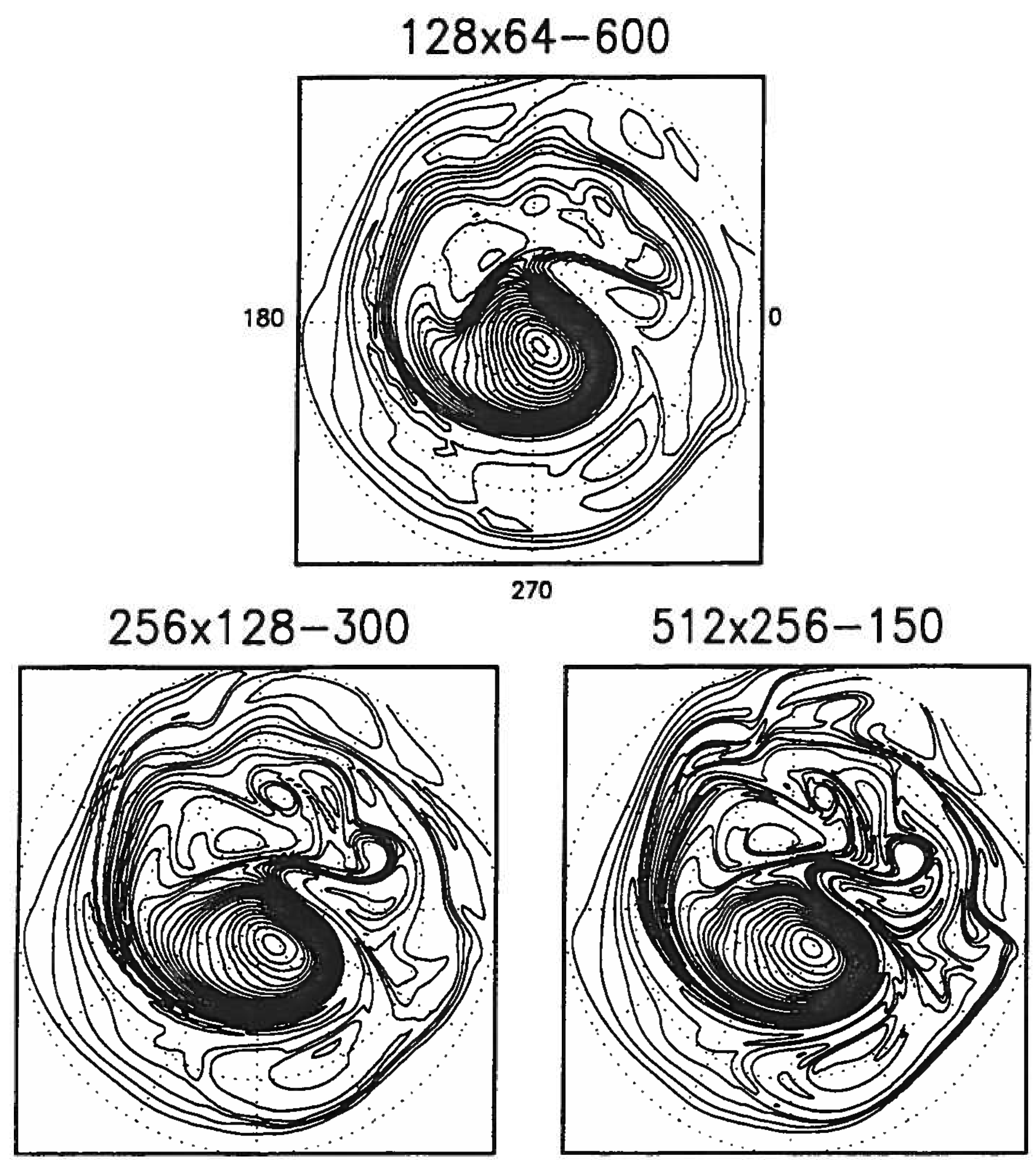


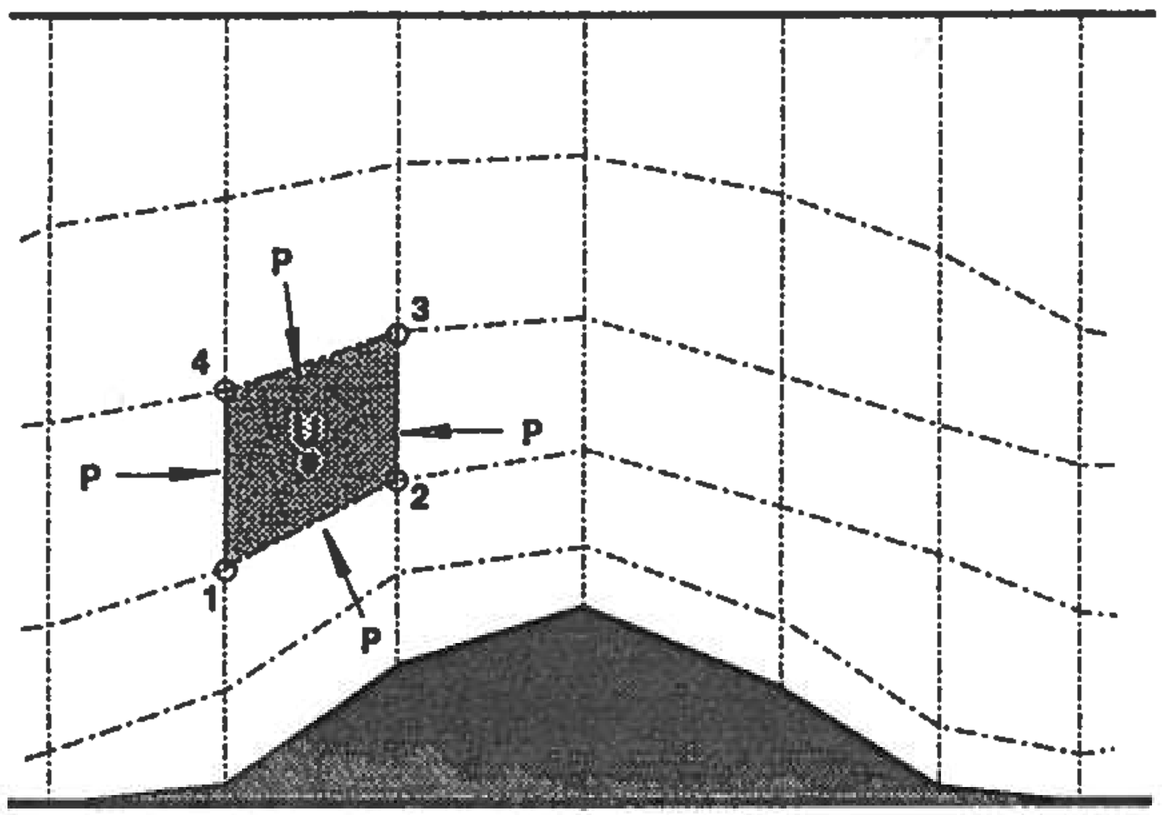
Figure 10. Polar stereographic projection (from the equator to the north pole) of the potential vorticity contours at DAY-24 in the 'stratospheric vortex erosion' test case at three different resolutions.

Development of the FV^3 core

- Lin and Rood (1996, MWR): Flux-form advection scheme
- Lin and Rood (1997, QJ): FV shallow-water solver
- **Lin (1997, QJ): FV Pressure Gradient Force**
- Lin (2004, MWR): Vertically-Lagrangian discretization
- Putman and Lin (2007, JCP): Cubed-sphere advection
- Harris and Lin (2013, MWR): Describes FV^3 and grid nesting
- Lin (in prep): Nonhydrostatic dynamics

Lin (1997, QJ)
Finite-Volume Pressure Gradient Force

- Computed from Newton's laws and Green's Theorem
- The pressure force on one cell from its neighbor is equal and opposite to that exerted by the cell on its neighbor
- Momentum is conserved the same way finite-volume algorithms conserve mass



$$\left(\frac{\mathrm{d}u}{\mathrm{d}t}, \frac{\mathrm{d}w}{\mathrm{d}t}\right) = \frac{1}{\Delta m}(\Sigma \mathbf{F}_x, \Sigma \mathbf{F}_z)$$

$$\Sigma \mathbf{F} = \int_C P\mathbf{n}\,\mathrm{d}s$$

$$\frac{\mathrm{d}u}{\mathrm{d}t} = g\frac{\Sigma \mathbf{F}_x}{\Sigma \mathbf{F}_z} = g/\tan\gamma$$

Lin (1997, QJ)
Finite-Volume Pressure Gradient Force

- Errors lower, with much less noise, compared to a finite-difference pressure gradient evaluation
- Linear line-integral evaluation used in example yields larger errors near model top
    - Now using fourth-order scheme to evaluate line integrals

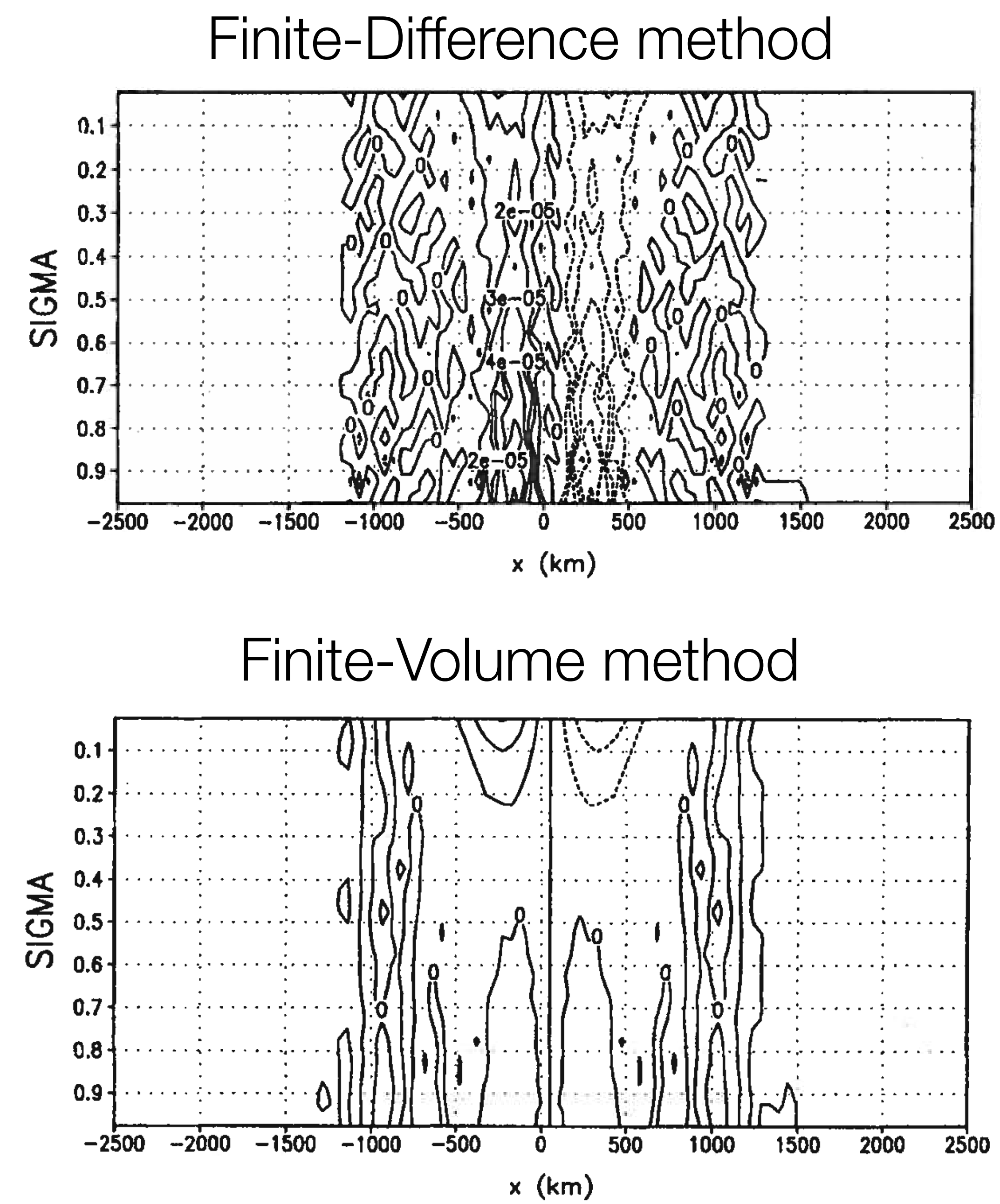
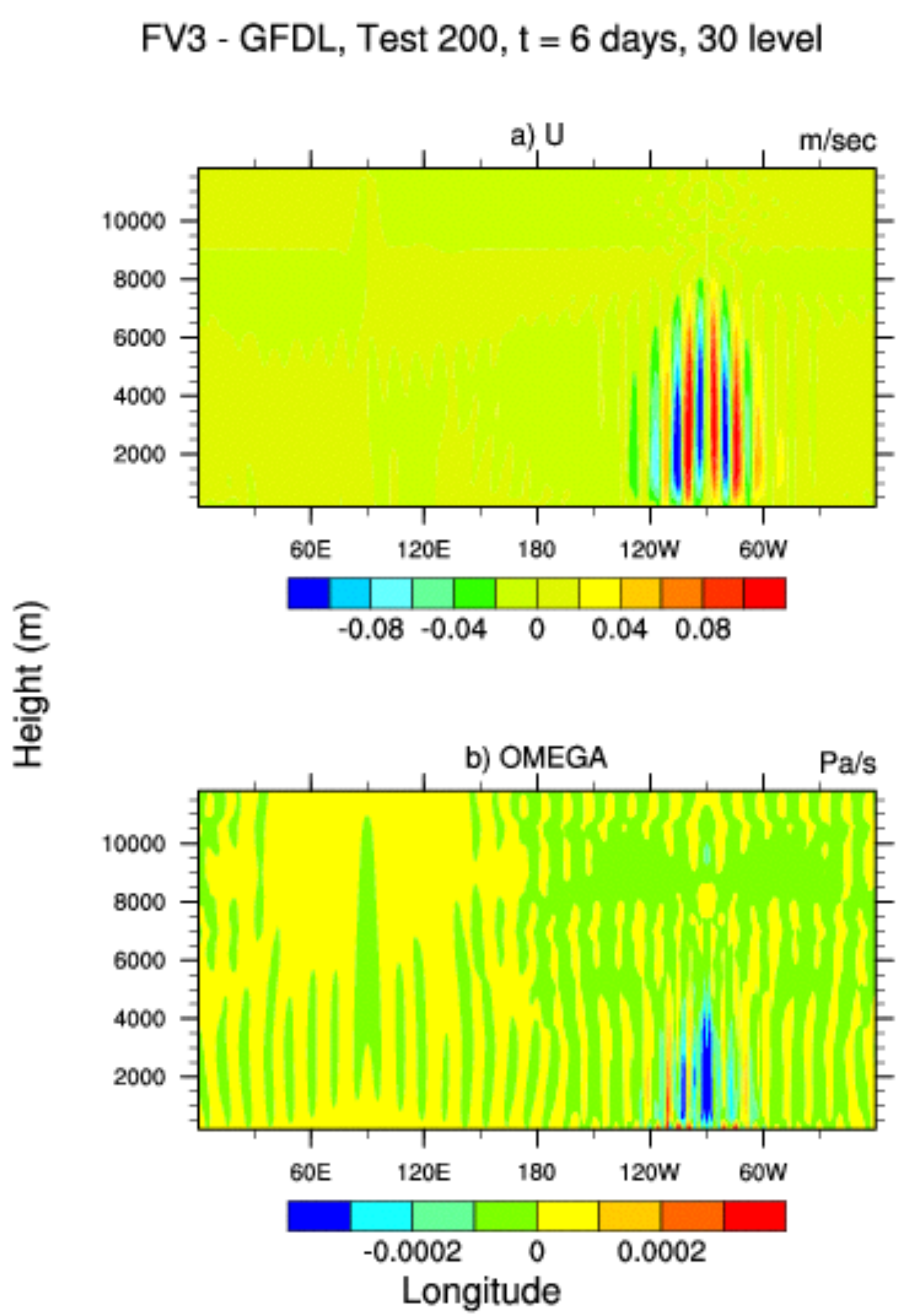


Figure 6. As in Fig. 5, but for the finite-volume method.

DCMIP 2012
Resting atmosphere test

- DCMIP Test 2-0-0
- 15 years later: same great results!
    - Compare to other DCMIP participants (links to DCMIP website):
      CAM-FV (lat-lon FV core)
      CAM-SE
      UKMO ENDGAME
      ICON IAP
      ICON MPI DWD
      MPAS



Development of the FV^3 core

- Lin and Rood (1996, MWR): Flux-form advection scheme
- Lin and Rood (1997, QJ): FV shallow-water solver
- Lin (1997, QJ): FV Pressure Gradient Force
- **Lin (2004, MWR): Vertically-Lagrangian discretization**
- Putman and Lin (2007, JCP): Cubed-sphere advection
- Harris and Lin (2013, MWR): Describes FV^3 and grid nesting
- Lin (in prep): Nonhydrostatic dynamics

Lin (2004, MWR)
Vertically-Lagrangian Discretization

- Equations of motion are vertically integrated to yield a series of layers
- Each layer like shallow-water, except θ is active
- Layers deform freely while horizontal equations integrated
  - Only cross-layer interaction here is through pressure force
- To perform vertical transport, and to avoid layers from becoming infinitesimally thin, we periodically **remap** to an Eulerian vertical coordinate

Vertical remapping

- Reconstructions by a cubic spline for remapping accuracy
  - Implicit in vertical, so no message passing
- Remapping conserves mass, momentum, and geopotential
  - Option to apply an energy fixer
- Vertical remapping is computationally expensive, but only needs to be done a few times an hour, **not every time step**
- As long as $\delta p > 0$, we retain stability. **No** vertical courant number limitation! This becomes critically important in nonhydrostatic simulations.

FV^3 and the GFDL models

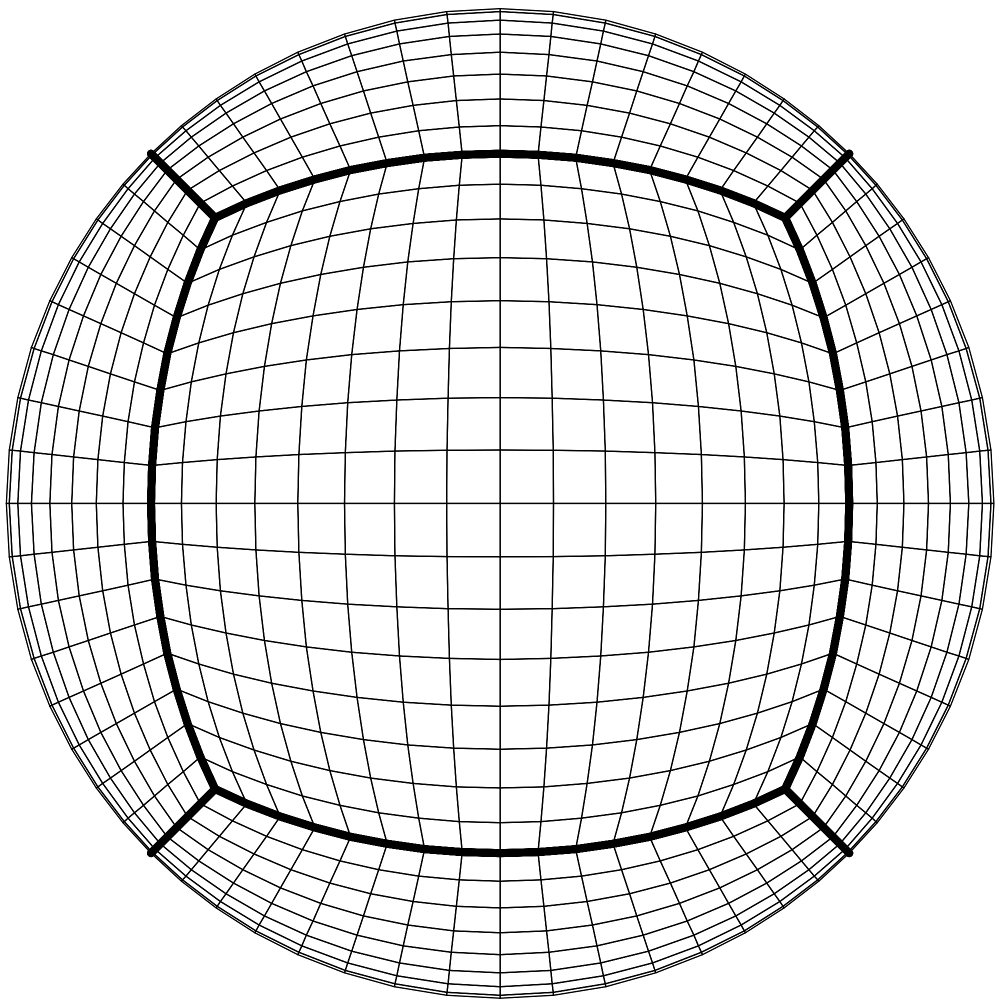
- Terrain following pressure coordinate: $p_k = a_k + b_k p_s$
    - Other coordinates possible: hybrid-z, hybrid-isentropic
- Divergence damping: fourth-order damping now standard, with a sixth-order option
    - Hyperdiffusion on vorticity also available. Useful when using non-monotonic schemes in very high-resolution nonhydrostatic simulations
- Physics coupling is time-split

Development of the FV^3 core

- Lin and Rood (1996, MWR): Flux-form advection scheme
- Lin and Rood (1997, QJ): FV shallow-water solver
- Lin (1997, QJ): FV Pressure Gradient Force
- Lin (2004, MWR): Vertically-Lagrangian discretization
- **Putman and Lin (2007, JCP): Cubed-sphere advection**
- **Harris and Lin (2013, MWR): Describes FV^3 and grid nesting**
- Lin (in prep): Nonhydrostatic dynamics

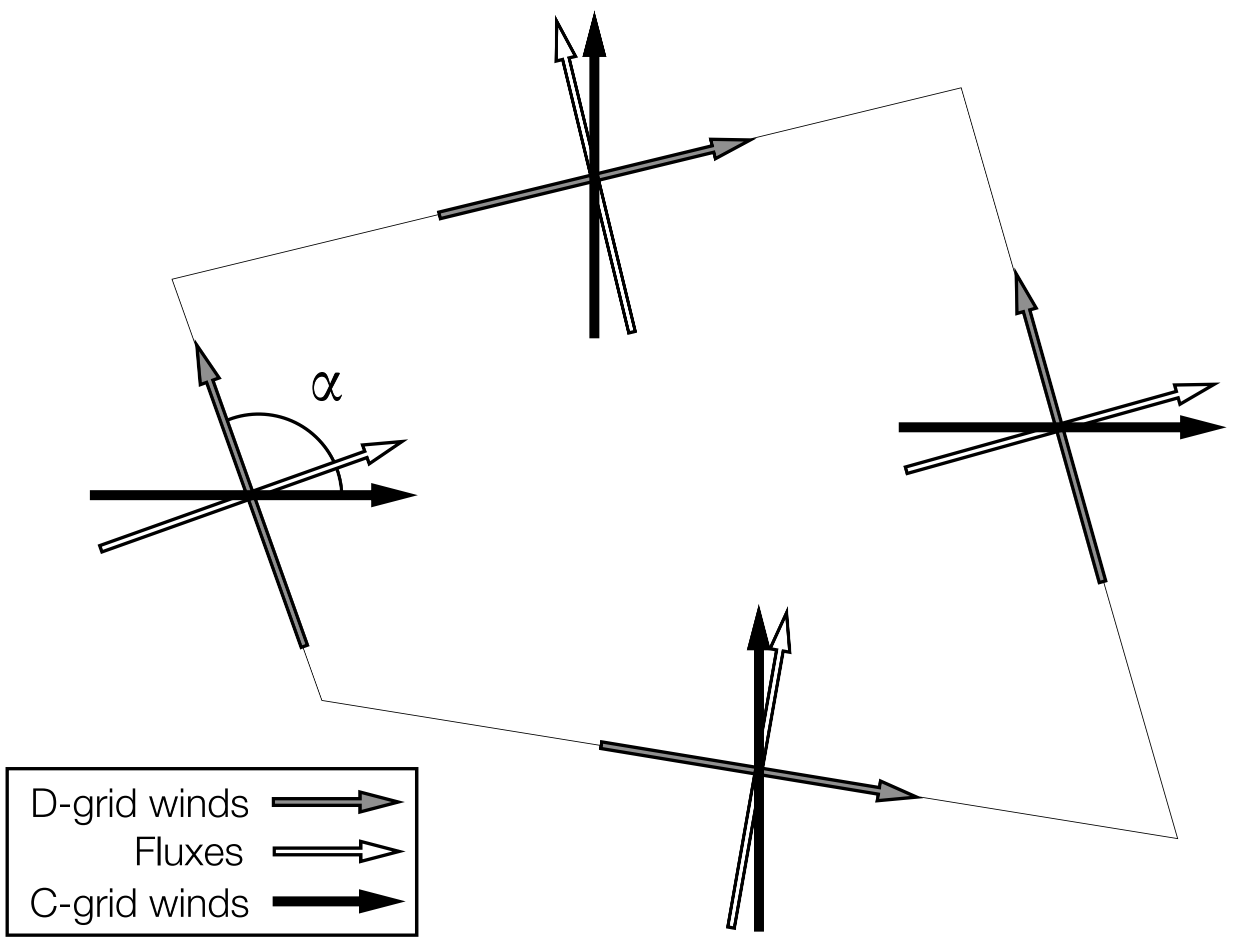
Putman and Lin (2007, JCP)
Cubed-sphere advection

- Gnomonic cubed-sphere grid
  - Coordinates are great circles
- Widest cell only √2 wider than narrowest
  - More uniform than conformal, elliptic, or spring-dynamics cubed spheres
- Tradeoff: coordinate is non-orthogonal



Putman and Lin (2007, JCP)
Non-orthogonal coordinate

- Gnomonic cubed-sphere is non-orthogonal
- Instead of using numerous metric terms, use covariant and contravariant winds
  - Solution winds are covariant
  - Advection is by contravariant winds
  - KE is product of the two



Cubed-sphere edge handling

- Fluxes need to be the same across edges to preserve mass-conservation
- Gnomonic cubed sphere has ‘kink’ in coordinates at edge
- Currently getting edge values through two-sided linear extrapolation
- More sophisticated edge handling in progress

Development of the FV^3 core

- Lin and Rood (1996, MWR): Flux-form advection scheme
- Lin and Rood (1997, QJ): FV shallow-water solver
- Lin (1997, QJ): FV Pressure Gradient Force
- Lin (2004, MWR): Vertically-Lagrangian discretization
- Putman and Lin (2007, JCP): Cubed-sphere advection
- Harris and Lin (2013, MWR): Describes FV^3 and grid nesting
- **Lin (in prep): Nonhydrostatic dynamics**

Nonhydrostatic FV^3

- FV^3 does a great job for hydrostatic flows. How can we maintain the hydrostatic performance and still do a good job with nonhydrostatic dynamics??

- Introduce new prognostic variables: w and δz (height thickness of a layer). The pressure thickness is still hydrostatic pressure, and thereby mass

- Density is then easily computed:

$$\rho = \frac{M}{V} = \frac{W}{gV} = \frac{\delta p \Delta A}{g \delta z \Delta A} = \frac{\delta p}{g \delta z}$$

- Nonhydrostatic pressure is then diagnosed as a deviation (**not** small-amplitude!) from the hydrostatic pressure.

Nonhydrostatic FV^3

- FV^3 does a great job for hydrostatic flows. How can we maintain the hydrostatic performance and still do a good job with nonhydrostatic dynamics??

- Introduce new prognostic variables: w and δz (height thickness of a layer). The pressure thickness is still hydrostatic pressure, and thereby mass

  - Vertical velocity w is the cell-mean value. Remember that vorticity is also a cell-mean value.

  - Helicity can be computed without averaging!

FV^3 **nonhydrostatic** solver: Time-stepping procedure

- Interpolate time t^n D-grid winds to C-grid
- Advance C-grid winds by one-half timestep to time $t^{n+1/2}$
- Use time-averaged air mass fluxes to update δp and θ, **and to advect w and $\boldsymbol{\delta z}$**, to time t^{n+1}
- Compute vorticity flux and KE gradient to update D-grid winds to time t^{n+1}
- **Solve nonhydrostatic terms for w and nonhydrostatic pressure perturbation using either a Riemann solver or a semi-implicit solver**
- Use time t^{n+1} δp, $\boldsymbol{\delta z}$, and θ to compute PGF to complete D-grid wind update

Nonhydrostatic FV^3: nonhydrostatic solvers

- Instead of using a time-split solver for the fast vertical waves, FV^3 presents two solvers for the nonhydrostatic terms:

  1. Exact Riemann solver: solves for the Riemann invariants along the gravity wave characteristic curves. Highly accurate!

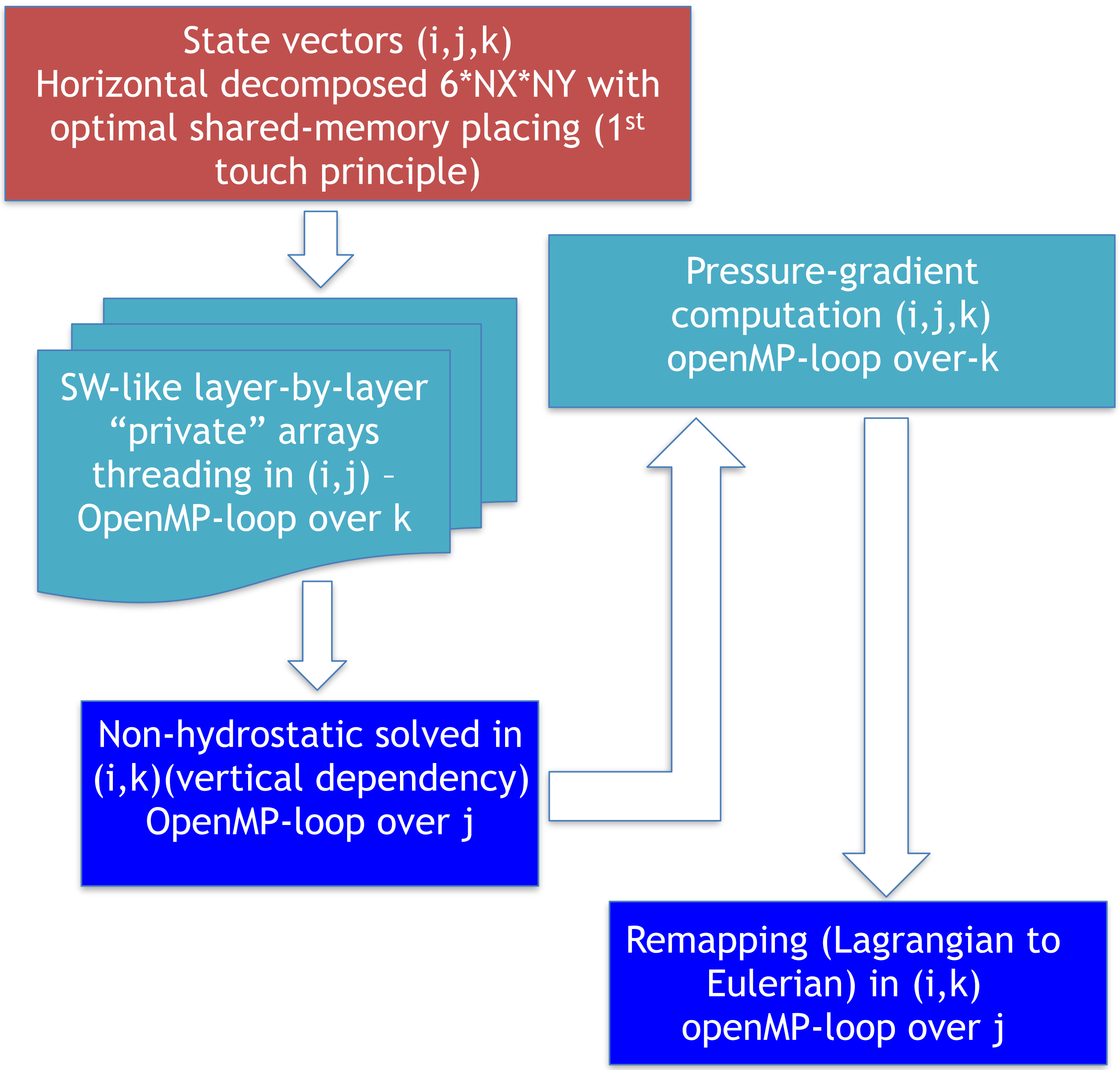
  2. Semi-implicit solver: solves a vertically-tridiagonal system for the sound waves. Diffusivity in semi-implicit solver works to damp sound waves.

- **Most simulations do very well with the semi-implicit solver.** For *very* high-resolution simulations ($\Delta x < 1$ km) where the vertical Courant number is < 1, the Riemann solver may be more appropriate (and possibly faster).

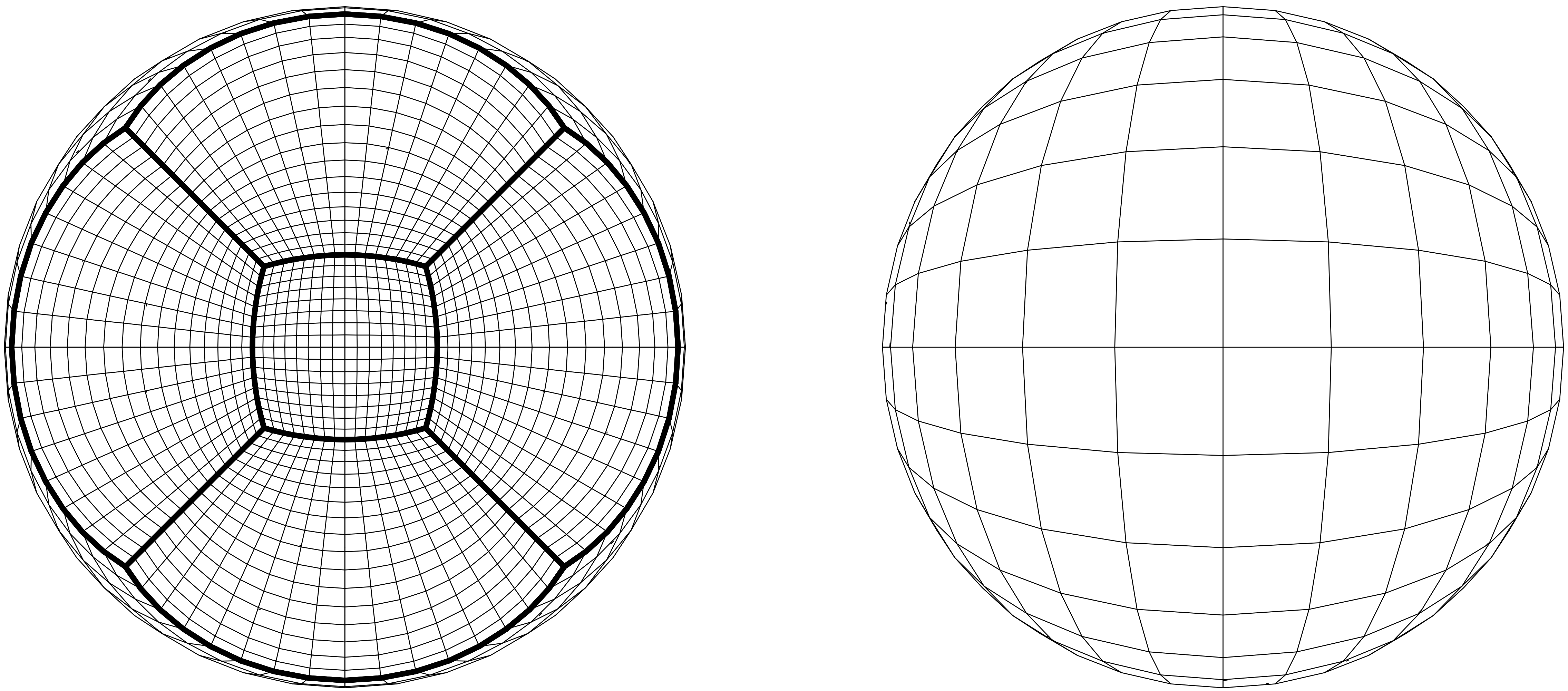
  - **All FV^3 simulations for NGGPS use the semi-implicit solver**

FV^3: Model design and model performance

- Scientific accuracy is very important. But performance considerations **cannot** be ignored
- FV3 originally designed for 90's vector supercomputers: lots of concurrency with a minimum of copies and transposes.
- Shared-memory threading and distributed-memory decomposition work together—not against one another!

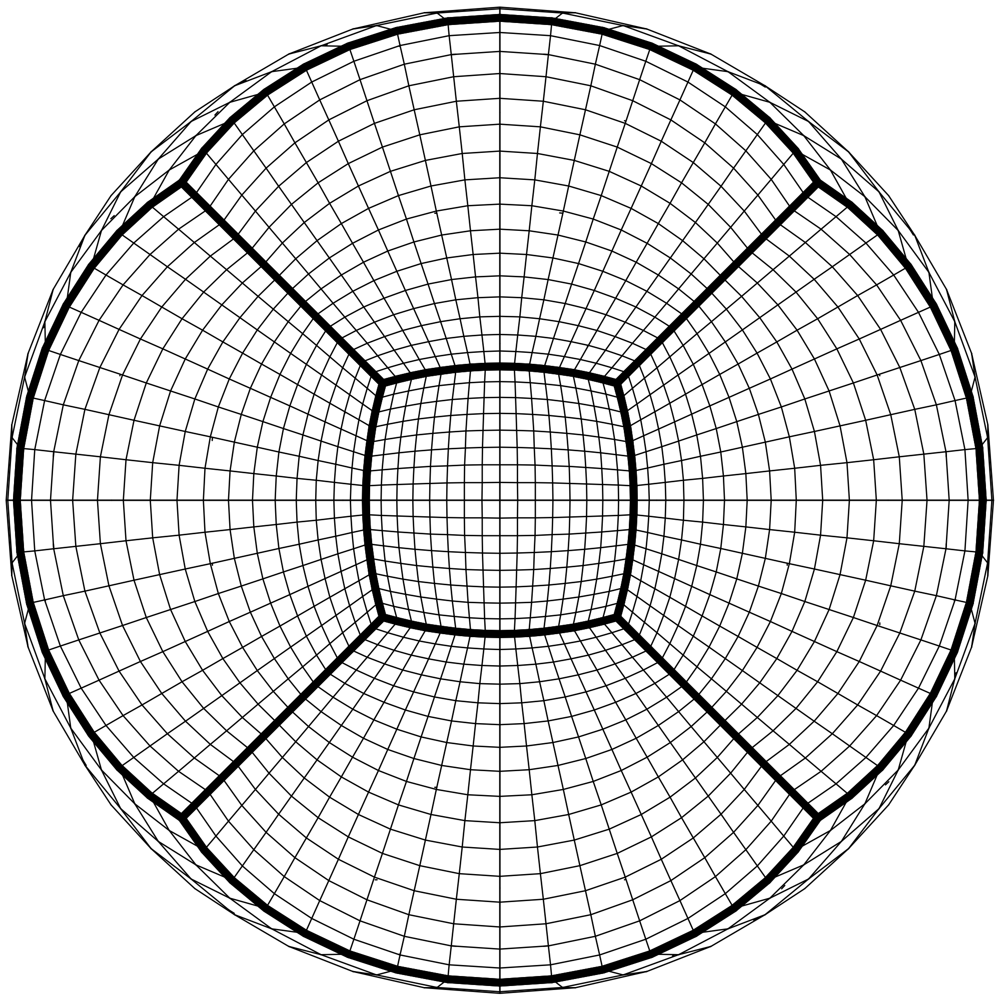
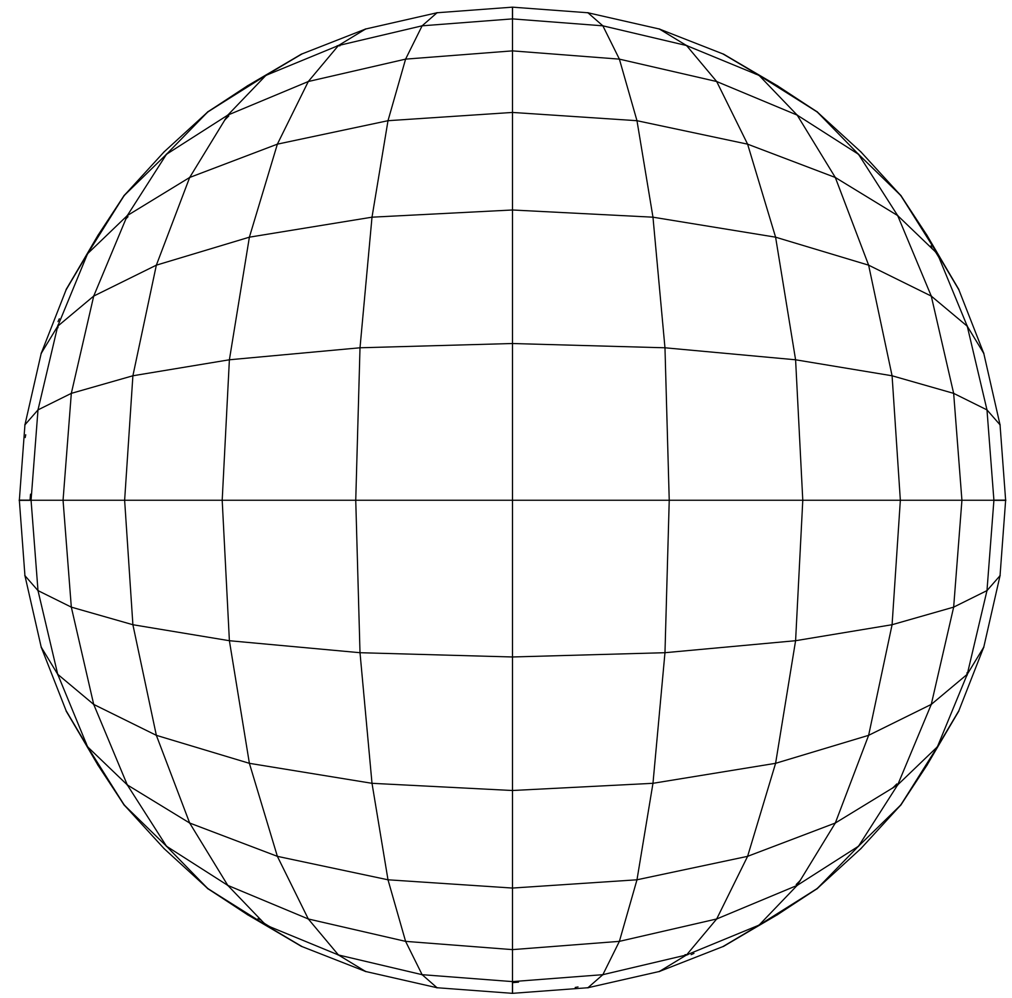


Stretched grid



- Deforms a global grid so that one face has a higher resolution than the others
- Conceptually straightforward: requires no changes to solver!

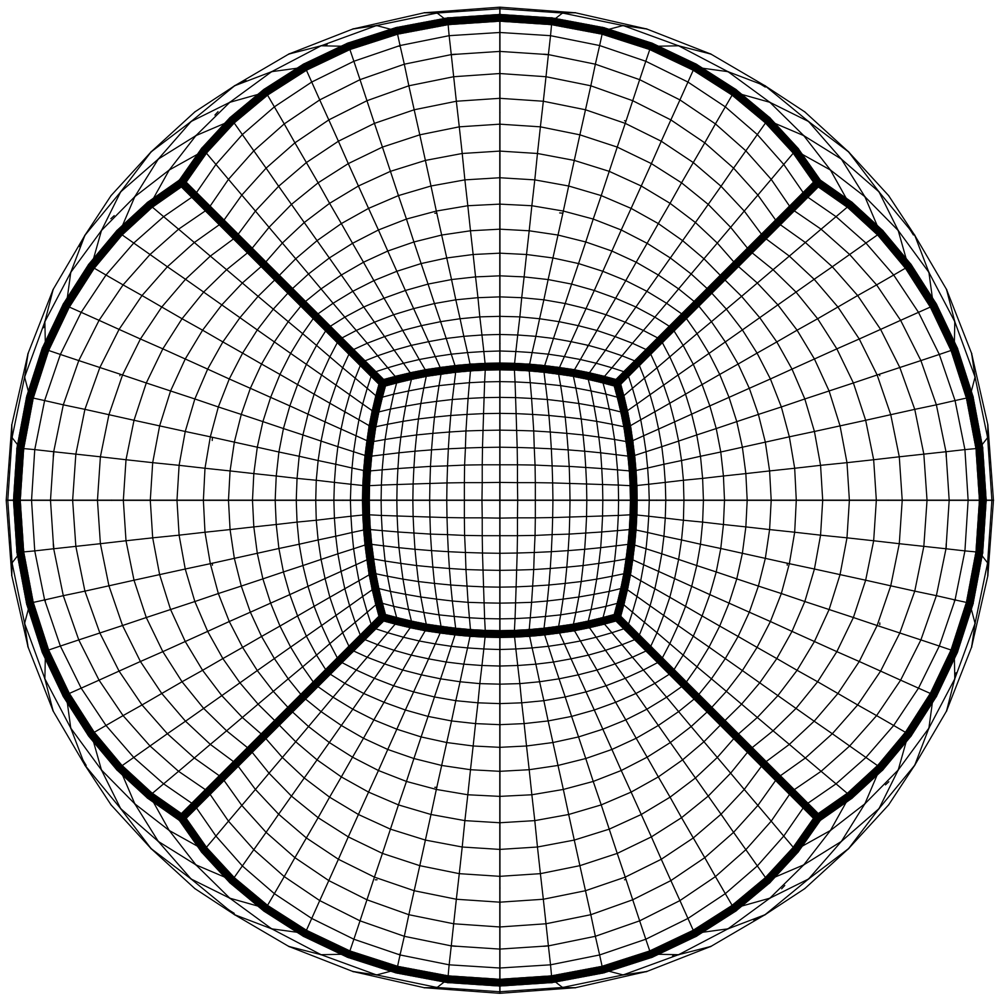
Stretched grid

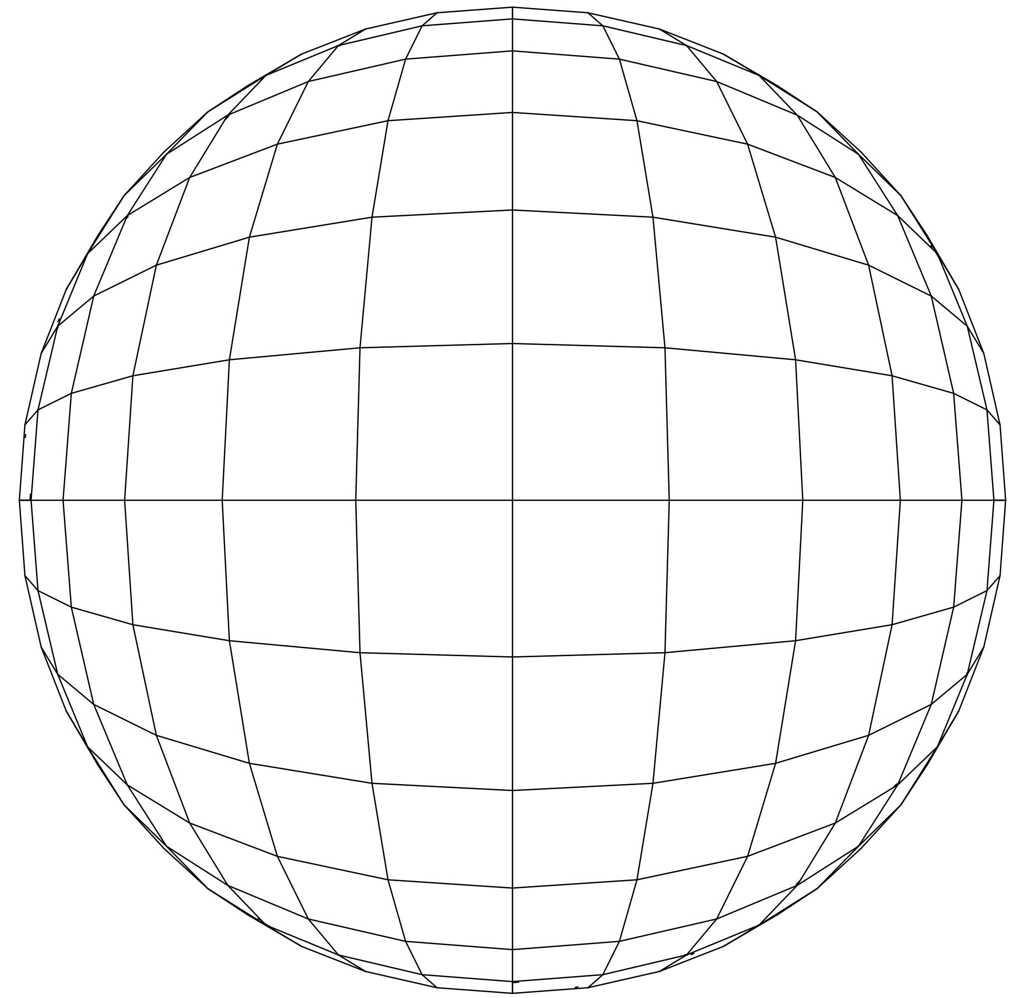



- Smoothly deformed! Even continental-scale flows may see little effect from the refinement.

- Capable of **extreme** refinement (80x!!) for storm-scale simulations

Stretched grid

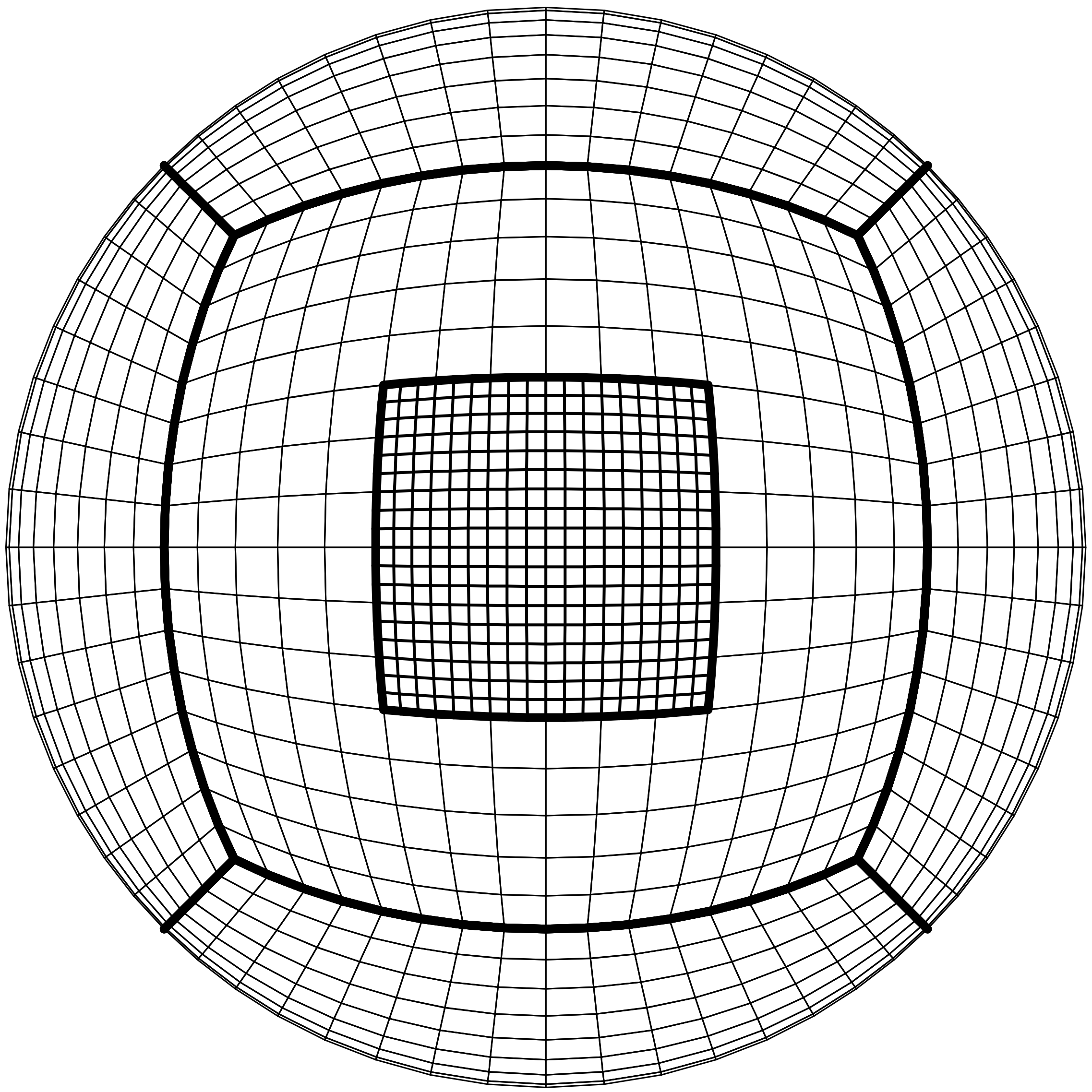


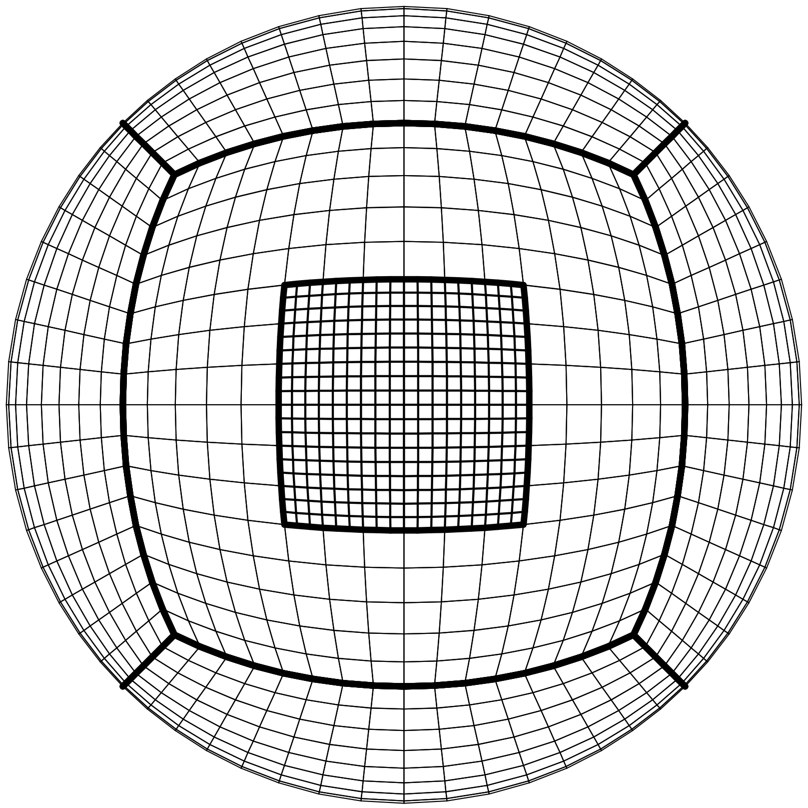


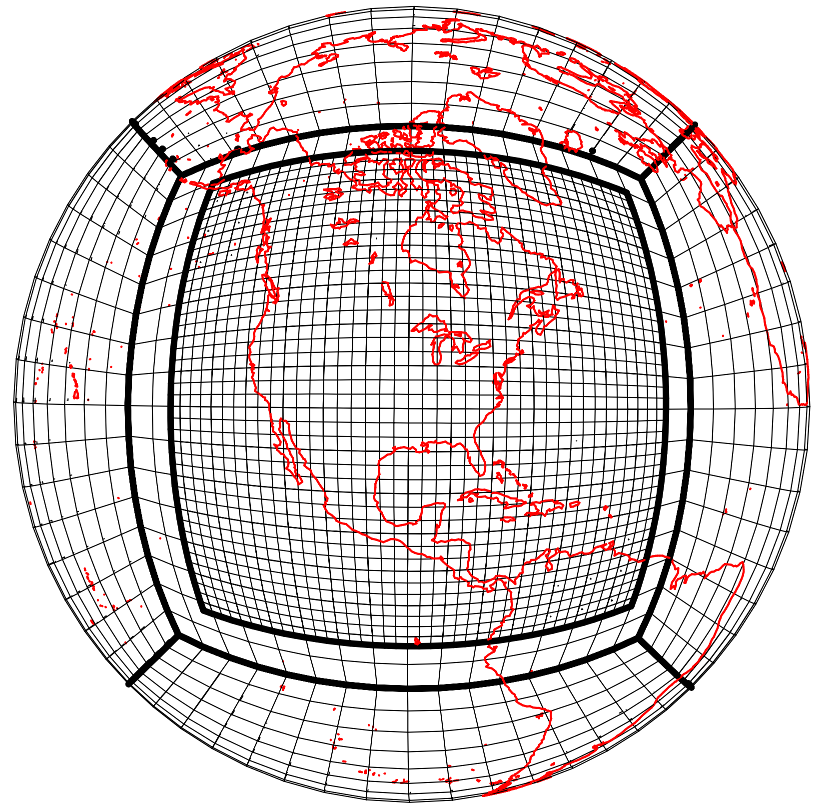
- Opposing face can become **very** coarse
- Scale-aware parameterizations? (Are these really so important?)

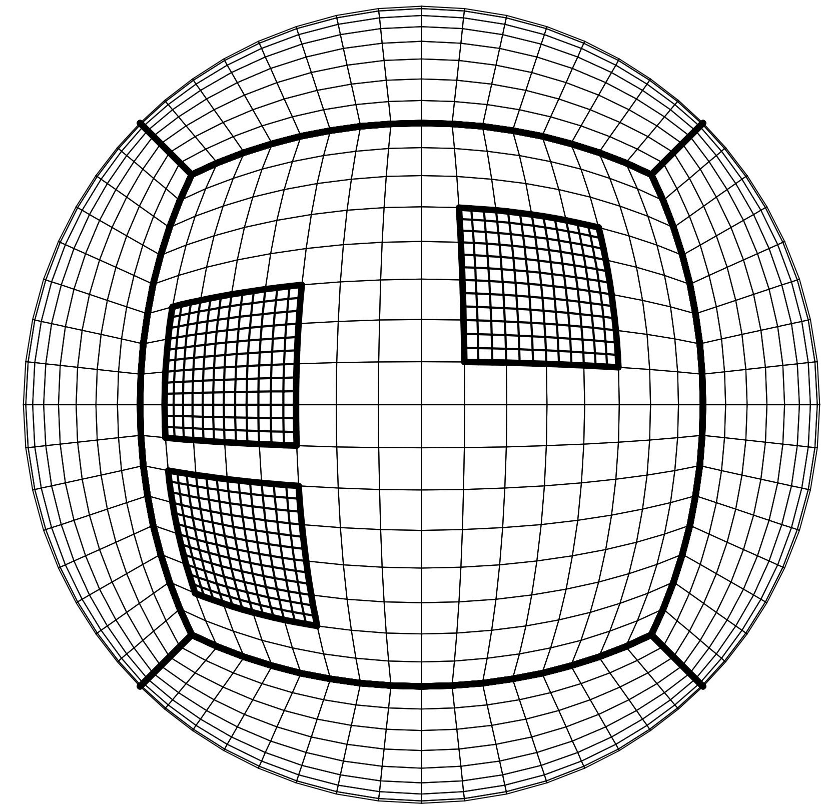
Grid nesting

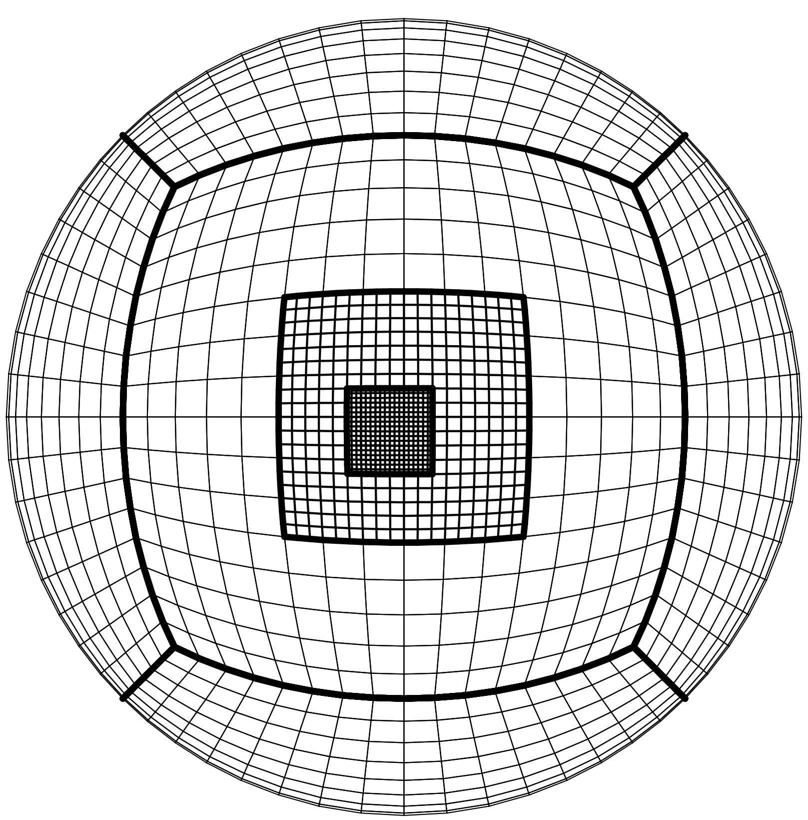
- **Simultaneous** coupled, consistent global and regional solution. No waiting for a regional prediction!
- Different grids permit different parameterizations; **doesn't need a "compromise" or scale-aware physics** for high-resolution region
- **Very flexible!** Currently only uses static nesting but moving nests are also possible

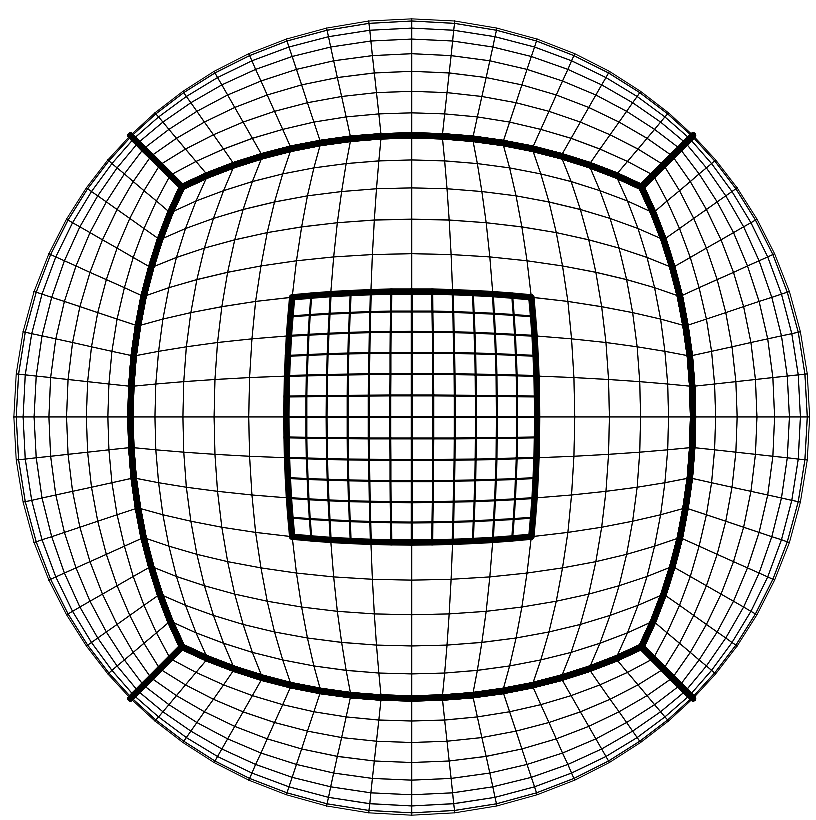


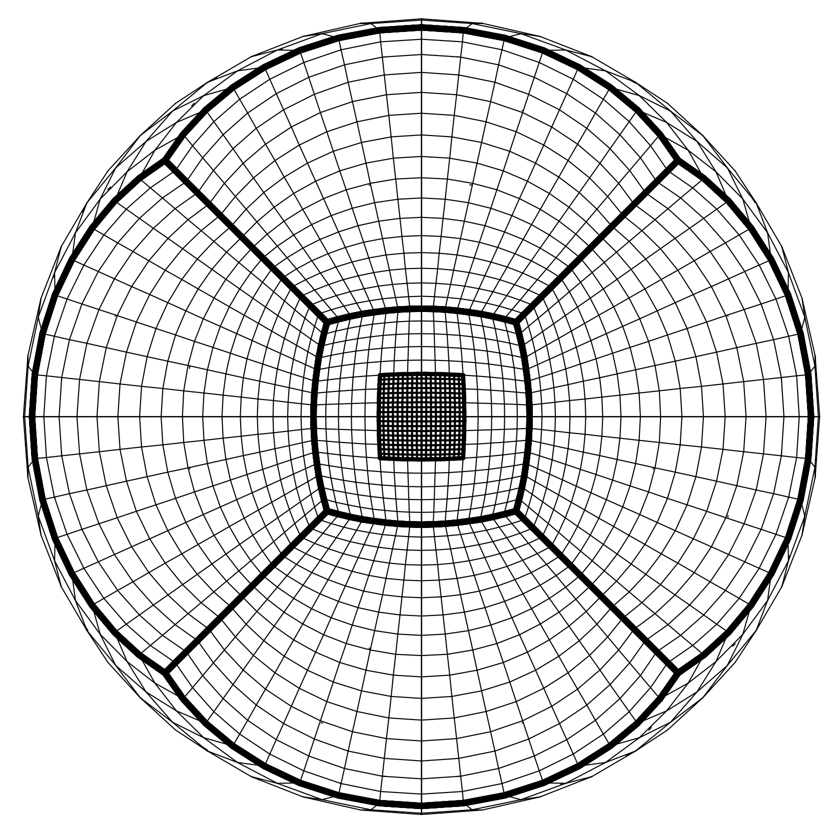

3:1 nested grid


Large nest for RCMs


Multiple nests


Telescoping nests


2:1 nested grid

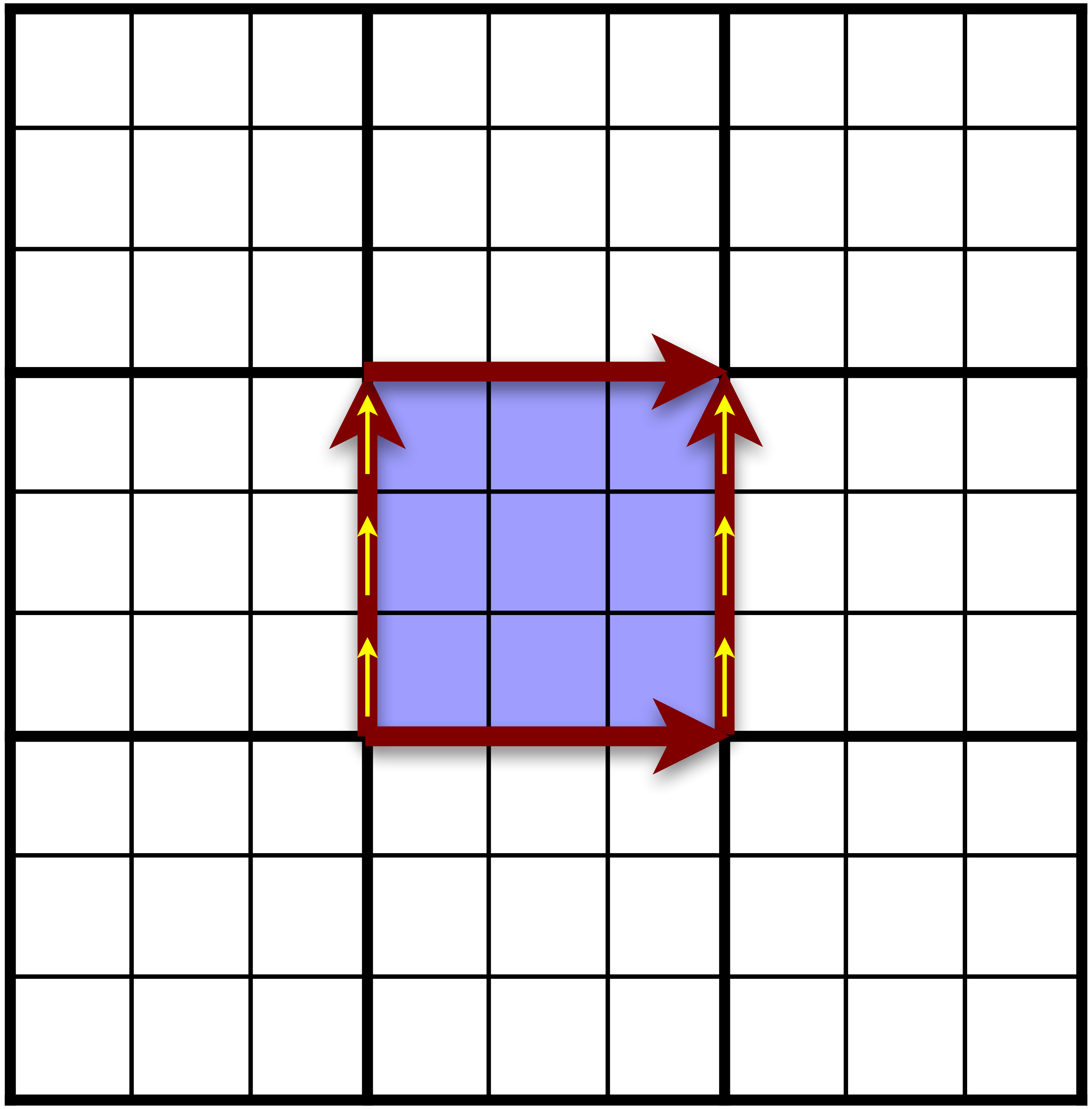

Nest in stretched grid

Nesting methodology: boundary conditions

- Simple interpolation BC
  - Linearly interpolate all variables in time and space to fill nested-grid halo
  - Traditionally, time interpolation requires the coarse grid to be advanced before the nested grid
- **Concurrent nesting**: integrate coarse and nested grids simultaneously by **extrapolating** coarse-grid data in time to create nested-grid BC
  - For scalars: extrapolation is limited so that the BCs are positive-definite
- Nonhydrostatic nesting: use same solver to produce BCs for the (diagnosed) nonhydrostatic pressure perturbation. Treat w and δz as if they were the other variables

Nesting methodology: two-way update

- Simple averaged update
    - Cell average on scalars
    - In-line average for winds, to conserve vorticity
- Averaging is more consistent with FV discretization than pointwise interpolation

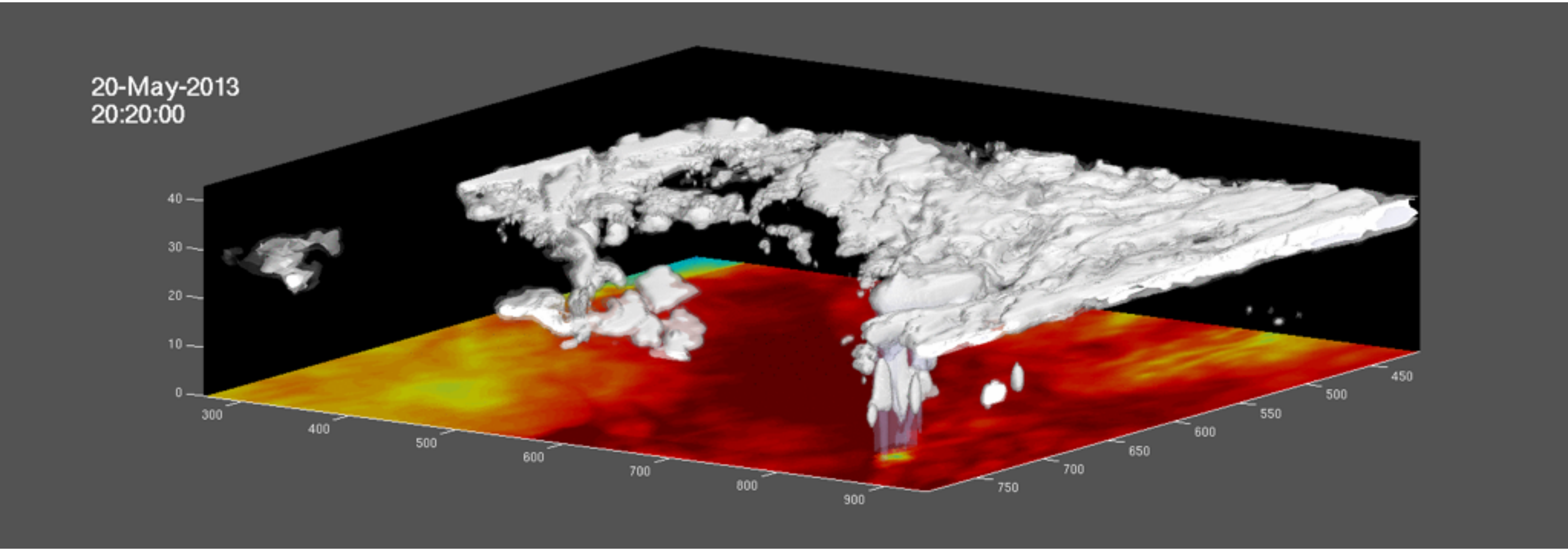


Mass conservation two-way nesting

- Usually quite complicated: requires flux BCs, conserving updates, and precisely-aligned grids
- Update only winds and temperature; not δp or δz
  - Two-way nesting overspecifies solution anyway
- **Very simple**: works regardless of BC and grid alignment

★ δp is the vertical coordinate: need to remap the nested-grid data to the coarse-grid's vertical coordinate

- Option: a "renormalization-conserving" means of updating tracers to the coarse grid while conserving tracer mass



Nonhydrostatic HiRAM
2013 Moore Outbreak
72-hour forecast
1.3 km nest