



**Western Region Technical Attachment**  
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**HOW DO YOU TRAP A RADAR BEAM?**

Every radar operator has at one time or another seen echoes affected by anomalous propagation. Under certain atmospheric conditions, a radar beam which would normally bend upward (relative to the earth) can bend downward and hit the ground, or bend just enough to follow the curvature of the earth and reach very distant targets. Such conditions usually happen at night when nocturnal inversions are present and when layers of thunderstorm outflow are near the surface. But what strength of an inversion is needed to trap a radar beam? And will all beams be trapped in such a layer? In this Technical Attachment (TA), we examine some of the issues involved in bending radar beams.

The bending of any electromagnetic wave passing through the atmosphere (like a radar beam) is produced by changes in the refractive index ( $n$ ) in the atmosphere. If we ignore the effects of free electrons (which should only be important in the ionosphere) and assume that the radar has a wavelength around 10cm, then the index of refraction is related to the pressure, temperature, and vapor pressure of the air by the following formula:

$$n = \left( \left( \left( 77.6 \frac{P}{T} \right) + \left( 373256 \frac{e}{T^2} \right) \right) \times 10^{-6} \right) + 1.0 \quad (1)$$

where  $e$  and  $P$  are in millibars and  $T$  is in Kelvin. The values of 77.6 and 373256 have been determined empirically, and vary somewhat in the literature, but these represent reasonable values. From an atmospheric sounding, we have values of pressure, temperature and vapor pressure at different heights and, thus, we can calculate the changes in refractive index with height. If we know the angle of the radar beam (relative to horizontal) and denote it  $\theta$ , we can use the change of the refractive index with height to calculate how the angle of the radar beam changes (relative to the earth) as it passes through the layer, by using the following formula:

$$\frac{\partial \theta}{\partial S} = \frac{1}{R} + \frac{\partial n}{\partial H} \quad (2)$$

Where  $S$  is the distance travelled through the layer,  $R$  is the radius of the earth. If we assume a spherical earth with a radius of 6374km (which is not always a good assumption, but the subject of another TA), then the values of refractive index ( $n$ ) have to decrease by more than  $4.782 \times 10^{-5}$  per 1000 ft ( or  $1.569 \times 10^{-4}$  per km ) to bend a horizontal beam downward. It is relatively easy to calculate the gradient of  $n$  with height (or more commonly the gradient of  $N$  which is defined as  $(n-1) \times 10^6$  ) based on raob data, and, thus, we can calculate if a radar wave will bend upward or downward (relative to the earth) in any layer.

One thing to keep in mind is that the n-gradient only tells us the **rate of change** of the beam angle as it passes through the layer. If the beam enters the layer at a high angle, it may not be bent enough to stay within the layer before exiting out the top. Thus, a layer with an n-gradient that might be labelled a "trapping layer" can be easily penetrated by beams that enter the layer at a high angle.

Another thing to keep in mind is that a radar beam has a finite width. For the WSR-88D, the beam width is around  $1^\circ$ . This means that for an elevation angle of  $0.5^\circ$ , where the center of the beam makes an angle of  $0.5^\circ$  with the horizontal, the bottom of the beam makes an angle of  $0^\circ$  with the horizontal, and the top of the beam makes an angle of  $1.0^\circ$ . Since, as we discussed above, the ability to trap a beam is dependent on the angle that the beam enters the layer, it is easy for the bottom part of the beam to be trapped in a layer (since it starts off at a lower angle) while the top part of the beam makes it through the layer and continues upward. Such a condition is actually quite common, and can lead to very different beam spreading patterns at long ranges than might first be expected.

### **Atmospheric conditions leading to possible trapping**

An examination of equation 1 can give us an idea of what conditions will produce large decreases of n with height, and, thus, be susceptible to trapping of the radar beam. Since pressure always decreases with height, then an increase of temperature with height (an inversion) will cause the first term ( $77.6 P / T$ ) to decrease more with height than it would if the temperature were isothermal, or decreased at a normal lapse rate. So: inversions, if strong enough, could lead to trapping layers.

A strong decrease of vapor pressure with height (say a few millibars over a layer 50mb thick) can lead to a dramatic drop in n since the term ( $373256 e / T^2$ ) has such a large constant. This effect is even more dramatic if T increases in this same layer (an inversion) since temperature is squared in the denominator of this term, and the first term would also be decreasing more than usual in such a case (because of the inversion). Such conditions are common across the marine layer inversions along the West Coast, and in thunderstorm outflow boundaries in the west where evaporation in below-cloud rainshafts leads to outflows of moist, cool air below the drier, warmer air that makes up the below-cloud environment.

However, we still haven't considered **how strong** an inversion is needed, and **how much** vapor pressure decrease is needed to produce trapping. Unfortunately, since three variables (pressure, temperature, and vapor pressure) are involved, it is impossible to make any "hard and fast" rules about the strength of inversions that are needed. In fact, if you calculate the n-gradients based on equation 1, the results are not always what you might expect. We will examine two cases to illustrate beam trapping in some typical Western Region environments.

#### **Case #1) Arctic Layer in Montana**

Strong arctic inversions are common in Montana in the winter when arctic air masses plunge southward out of Canada. Figure 1 shows a relatively deep arctic air mass over Great Falls, Montana at 00 UTC, December 29, 1992, with the arctic front between 800 and 700mb. The height, pressure, temperature, dew point, and N values at all levels below 600mb are listed in Table 1. The inversion is strongest between 758 and 752mb where the

temperature increases  $13.2^\circ$ , and you might think that this is the layer where trapping is most likely. Table 2, however, shows values of  $\partial\theta/\partial S \times 10^7$  (equation 2), such that negative values indicate that the beam is being bent downward relative to the earth. Interestingly, it is the layer between 716 and 713mb where trapping is possible -- not the layer between 758 and 752mb! The vapor pressure drops significantly over this layer while the temperature decreases only slightly, and, thus, the refractive index ( $n$ ) drops dramatically.

In Fig. 2, we show the path of a simulated  $1.0^\circ$  beam at elevation angle  $0.5^\circ$  originating from a radar at 8900ft with an atmosphere defined by the Great Falls raob, and the path of a beam in a standard atmosphere. The three lines for each beam indicate the bottom, centerline, and top of the beam in each case. The vertical scale shows altitude (above mean-sea-level) while the horizontal scale shows the distance from the radar (in kilometers). The horizontal lines indicate the location of the raob significant levels. Since the beam starts out in the trapping layer, the bottom of the beam (which starts out at an angle of  $0.0^\circ$ ) is trapped, while the top and center of the beam (which start out at an angle of  $0.5^\circ$  and  $1.0^\circ$ , respectively) make it through (although at lower elevations than they would in a standard atmosphere). Figure 3 shows another simulated beam with  $1.0^\circ$  elevation angle originating from 3698ft (30 ft off the surface at Great Falls). Now the beam enters the trapping layer at a high enough angle that the beam is bent only slightly downward by the trapping layer.

## Case #2) Marine Layer along West Coast

Figure 4 shows a raob for Oakland, California at 12 UTC 20 October 1991 (during the Oakland Hills/Tunnel Fire). Tables 3 and 4 again show the corresponding level data, and  $\partial\theta/\partial S$  values for this case. Again, note that it is the layer between 996 and 980mb, where the vapor pressure drops dramatically, where the best possibility for trapping is found. Figure 5 shows a simulated  $0.5^\circ$  elevation angle beam for this case, starting at an elevation of 500ft. Again, in this case, the bottom of the beam is trapped in the marine inversion, while the top and center part of the beam easily make it out. Figure 6 shows a  $0.0^\circ$  elevation angle beam starting from an elevation of 1500ft. Under standard atmospheric conditions, the bottom of the beam would be bent upward enough to never reach the surface. However, with the marine layer at 800ft and below, the beam enters this layer and is bent downward and hits the surface at around 65km range. The center and top parts of the beam propagate in a nearly normal fashion. While much of this beam is unaffected by the trapping layer and may not hit any precipitation targets, the part that hits the ground will likely produce so much reflected energy that very strong echoes will be erroneously displayed.

## Conclusion

The trapping of a radar beam is very sensitive to the vertical distribution of temperature and moisture. Trapping can have a dramatic effect on beam spreading and beam height, especially at distant ranges. Conditions leading to trapping layers are quite common in the Western Region, but only with specific combinations of elevation angles, inversion heights, inversion strengths, etc., will the trapping actually occur. Even if only a small fraction of the beam is trapped within a layer, it can produce strong and unexpected radar returns if the trapped part of the beam hits a mountain or distant precipitation. PC software to estimate the beam heights for a particular raob (used to generate the figures in this TA) will soon be made available.

Fig. 1

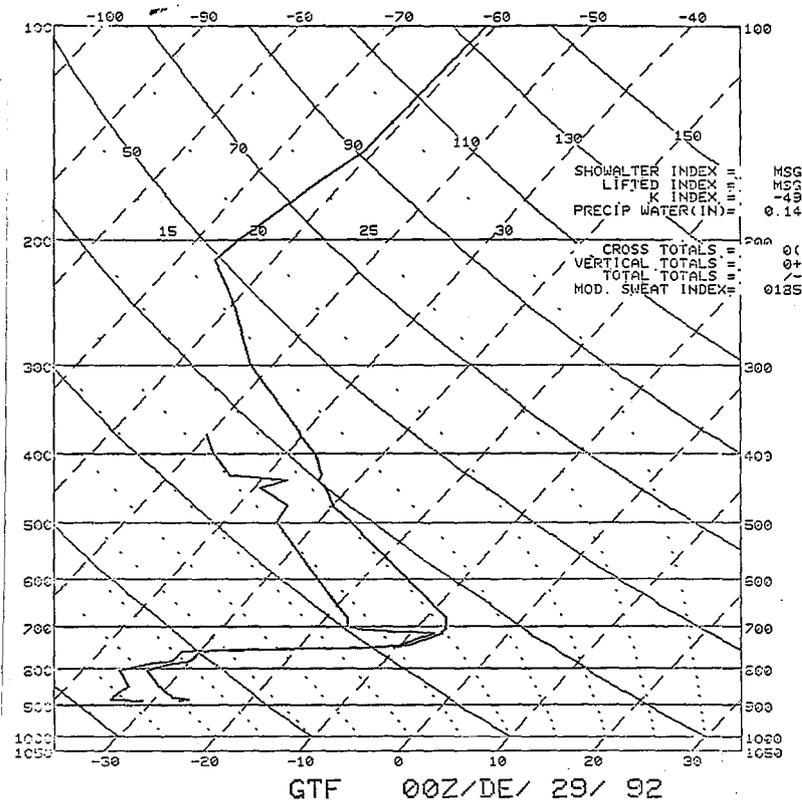


Table 1

Pressure (mb)	Height (m)	Temp. (°C)	Dewpoint (°C)	N-value
890	1118	-27.1	-31.0	282.8
886	1150	-26.3	-34.3	280.0
879	1207	-28.3	-34.3	280.0
850	1446	-30.7	-33.8	273.7
806	1837	-33.5	-36.3	262.2
791	1976	-31.9	-34.2	256.0
784	2041	-29.7	-31.7	251.9
761	2260	-29.7	-31.6	244.6
758	2289	-27.7	-29.5	242.1
752	2348	-14.5	-15.7	234.2
749	2377	-11.1	-12.2	233.4
744	2426	-8.3	-9.3	232.7
716	2709	-5.9	-6.8	225.9
713	2740	-6.1	-11.1	219.5
706	2812	-6.1	-15.1	213.7
700	2875	-6.3	-16.3	211.2
678	3115	-7.3	-17.3	204.9

Table 2

Pressure Layer (mb)	$\partial\theta/\partial S$ (rad/m)
890-886	0.6978
886-879	1.5831
879-850	1.3010
850-806	1.2769
806-791	1.1165
791-784	0.9434
784-761	1.2353
761-758	0.7191
758-752	0.2232
752-749	1.2774
749-744	1.4278
744-716	1.3274
716-713	-0.4920
713-706	0.7709
706-700	1.1737
700-678	1.3075

Fig. 2

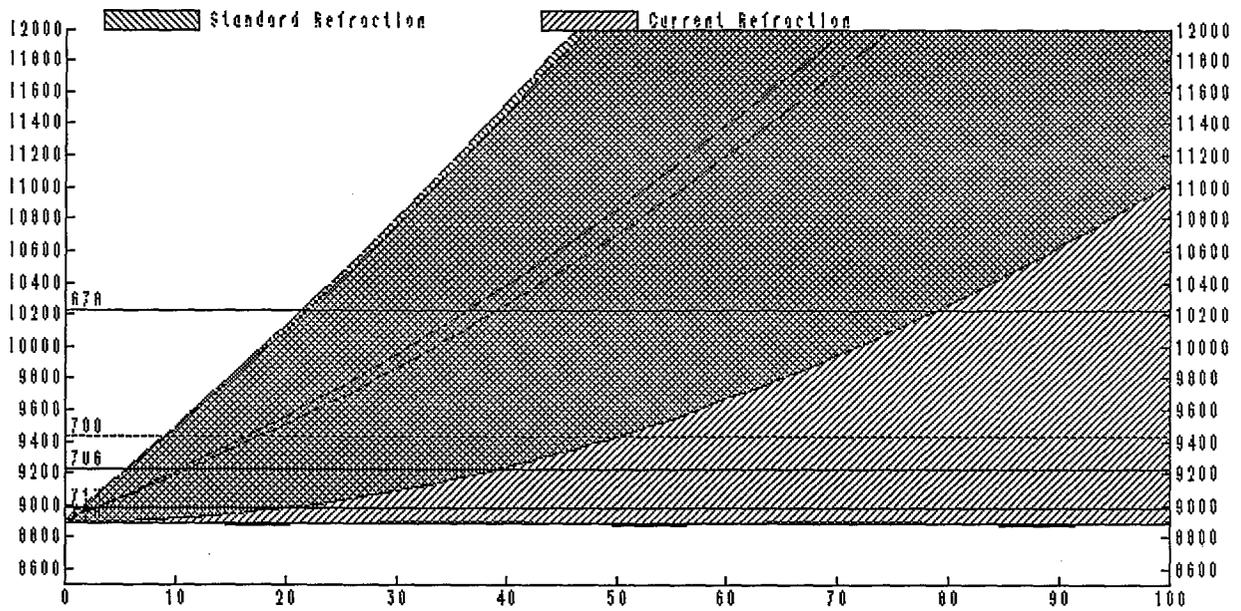


Fig. 3

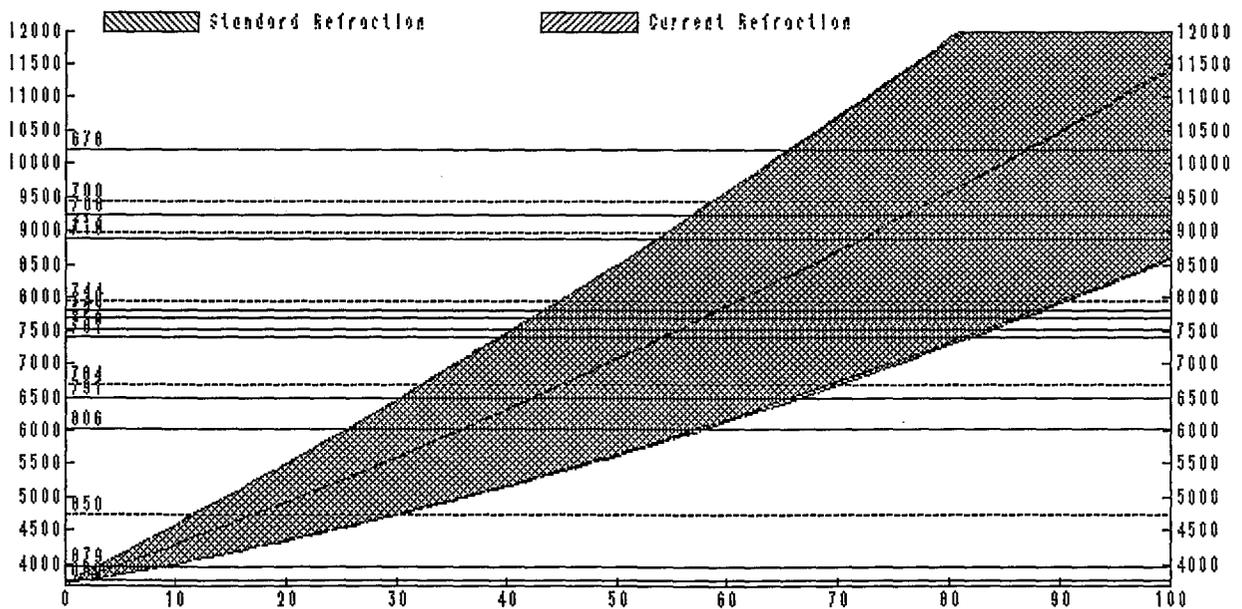


Fig. 4

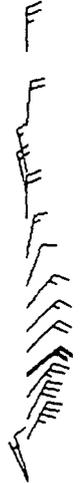
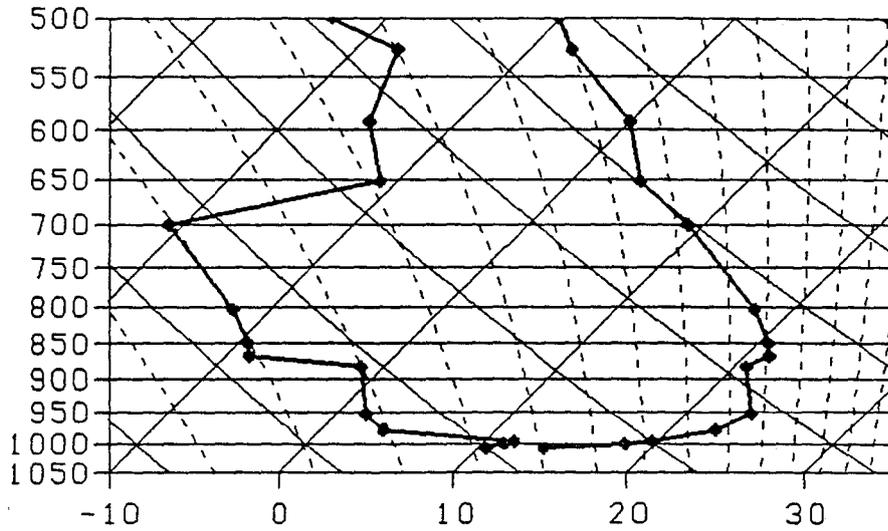


Table 3

Pressure(mb)	Height(m)	Temp.(°C)	Dewpoint(°C)	N-value
1009	2	13.8	10.4	330.0
1000	80	18.2	11.2	324.8
996	115	19.6	11.6	323.5
980	255	22.6	3.6	290.9
952	507	23.6	1.6	278.0
882	1169	20.6	-1.4	256.5
867	1317	21.4	-8.6	241.0
850	1489	20.6	-9.4	236.0
803	1970	17.8	-12.2	223.5
700	3130	9.2	-20.8	196.8
652	3698	4.0	-11.0	194.1

Table 4

Pressure Layer (mb)	$\partial \theta / \partial S$ (rad/m)
1009-1000	0.9040
1000-996	1.1844
996-980	-0.7551
980-952	1.0570
952-882	1.2444
882-867	0.5282
867-850	1.2971
850-803	1.3022
803-700	1.3386
700-652	1.5204

